ANALYTICAL METHODS OF PREDICTING GROUND EXCITATION FROM KNOWN RESPONSE

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

BY
SHARAT KUMAR SINHA

to the

DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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December 1970

POST GRADUATE OFFICE
Submitted on. 18/12/70

CERTIFICATE

Certified that this work on 'Analytical Methods of Predicting Ground Excitation from known Response' has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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Dated. 14.1.71

ABSTRACT

Analytical methods to solve the inverse analysis problem of predicting ground excitation from the known response of structures are presented. The dynamic response is assumed to be linear, damped and the result of unknown ground excitation. The unknown ground acceleration is idealized to be piecewise linear between points of sharp slope changes. Algorithms to predict both the time history of ground displacement and ground acceleration from the known time history of displacement or acceleration response of the structure are developed. Numerical experimentation on one and two degrees-of-freedom systems shows the stability and efficiency of the suggested algorithms. Slow as well as strong ground excitations are predicted accurately by these methods. A rigorous error analysis shows that the propagation of error in the suggested algorithm is relatively less as compared to finite difference method. The traditional method used to solve the response problem from the known ground excitation fails to perform the inverse process. This is discussed in an appendix together with its error analysis. Flow charts and computer programmes along with the user's guide are given in the appendices.

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CHAPTER 1

INTRODUCTION

The chief interests in structural dynamics problems are excitation, the system involved, and the response of the system to the excitation. The relationship between the cause and effect or excitation and response depends on the elastic and inertial characteristics of the physical system involved. The above three parameters lead to identification of three general class of problems in terms of the excitation, E, the system, S, and the response, R.

(a) Given the time history of the excitation, E, and the structural system, S, as described by its static and dynamic characteristics, what is the time history of its response, R ?

(b) Given the structural system, S, and response, R, what is the excitation, E ?

(c) Given the excitation, E, and the response, R, what are the design characteristics which define the structural system, S?

The source of excitation may be due to a large class of dynamic disturbances that differ both in form and origin. However, in the present study, the motion of the base of the structural systems have been considered as the source of excitation. Likewise, the response parameters can be displacement, velocity, acceleration, shear or any other quantity of interest. In the present work, only displacements and accelerations are taken as response quantities of interest.

The problems as grouped above can be respectively identified as (a) analysis, (b) inverse analysis, and (c) design aspects of the structural dynamics problems. A considerable volume of work has already been done in the field of analysis and design. However, very little effort seems to have gone into the inverse analysis aspect. The present study addresses itself to the problem of inverse analysis which deals with the prediction of time history of the excitation from the knowledge of physical properties of the structure, and the known time history of a response parameter. This, of course, calls for precision in the type of measuring instruments, namely, the displacement meters and accelerometers used to record response of structures relative to ground.

Any rational computer analysis of structures requires certain idealizations. One such idealization is to model an infinite degree-of-freedom continuous structures to a finite degree-of-freedom discretized equivalent system. The dynamic analysis of a n-degrees-of-freedom system can be performed by

(a) Modal superposition technique, and

(b) Simultaneous integration of n-coupled differential equations of motion.

In the present work, modal superposition method has been used to transform a n-degrees-of-freedom system to n single degree-of-freedom systems and representing the response of n-degrees-of-freedom system as a linear combination of the responses of n single degree-of-freedom systems.

The methods of dynamic analysis of a single degree-of-freedom can be grouped as:

(i) Methods employing numerical differentiation and integration.

(ii) Transform techniques.

(iii) Analog Computer methods.

(iv) Method leading to closed form solution if idealized forcing functions are used.

In general, the choice of techniques to solve the inverse analysis problem is dictated by the measured response parameter, the excitation parameter of interest, and the formulation of the problem.

Methods of the first category are, often, not useful to solve the inverse analysis problem due to the fact that numerical differentiation would be a 'noise' magnification process and any kind of error e.g. measurement, digitization, truncation, or round off can be disastrous. At the same time, numerical integration is a time consuming process and should not be resorted in the absence of mathematical knowledge of the forcing function.

Transform techniques once again are not of any worthwhile use, unless the closed form representation of the response gquantities are available.

Use of Analog Computer is often unreliable as the errors are attenuated in course of differentiation.

Method leading to closed form solution has been preferred because the idealized ground acceleration time history render the problem amenable to mathematical analysis and at the same time keeps the prediction close to the actual one.

1.2 Historical Background:

Ever since the introduction of modern strong motion seismograph in 1933 many investigators have tried to predict the response of certain mathematically idealized physical structures subjected to the motion of the base of the structure as recorded by the seismograph. Biot[1*] formulated the problem of shearing oscillation of buildings and gave the closed form expression for the time history of the displacement response. Some investigators devised mechanical analyzers for the response analysis of structures subjected to the ground motion. The advent of electronic digital computer and its capability to handle large volume of data led to the widespread use of digital computer in solving structural dynamics problems. Hershberger[2] discussed the effects of the shifts in accelerometers and showed how severe it was; thereby emphasising the use of Carder displacement meters. Schiff and Bogdanoff[3] has also discussed the effect of accelerogram vagaries, the center line adjustment, chart paper distortion. Schenker[4] came out with a method for determination of the response of tall structures like tier buildings and chimneys subjected to lateral forces or displacements. The above approaches relied on the accurate numerical evaluation of integrals. Recently, Nigam and Jennings[5] came out with a powerful method which avoids the use of numerical integrations. It has been shown there that the scheme is faster

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* The superscript numbers refer to the list of references.

than that utilizing numerical integrations. This method, reported for single degree-of-freedom system, is based on exact analytical solution of the governing differential equation of motion for the successive linear segments of the idealized ground acceleration time history. However, this method can be easily extended for multi degree-of-freedom systems using the principle of superposition of modes. Lately, Blume[6] has given a Spectral Response Reconciliation (SRR) procedure to obtain damping or other data under actual response to ground motions. A survey of these developments leads to the realization of the fact that no method is presently available for prediction of actual excitation from the known response of structures.

1.3 Aim, Scope and Limitations:

Development of algorithms to predict idealized ground excitation time history from the response history of a given structure is the aim of the present work. Such algorithms are particularly useful to analytically predict the ground shock in areas where instrumental means to obtain this information do not exist.

The developed algorithms have been tried on simple shear buildings. However, there seems to be no limitation for these being applied to any discretized complex structure.

In the absence of any response record available for a known structure, the simple structures chosen were analyzed for known ground shocks of both slow and strong motion type and then these were taken as inputs to the developed algorithms to reproduce these ground shocks.

CHAPTER 2

MATHEMATICAL DEVELOPMENT

The first step in any analytical study is the development of the mathematical equivalent of the physical system from the known properties of the structure. The complex nature of problems in structural dynamics calls for some idealizations. One such idealization is to model an infinite degree-of-freedom continuous system to a finite degree-of-freedom discretized system. The set of assumptions made during mathematical modelling in the present work are:

(i) The masses are considered to be lumped at certain discrete points on the structure.

(ii) The rotary inertia of the masses are neglected.

(iii) The masses are interconnected with massless elastic members.

(iv) The viscous damping coefficients are linear combinations of respective mass matrix and stiffness matrix coefficients.

2.1 Basic Equations of Motion:

(a) Single degree-of-freedom system:

The displacement time history of the viscously damped single degree-of-freedom system, Figure 1, subjected to the ground excitation is given by

$$m \ddot{x}(t) + c \dot{x}(t) + k x(t) = -m \ddot{x}_g(t) \tag{2.1}$$

where

m is total lumped mass,

c is the damping coefficient,

k is restoring stiffness,

$x_g(t)$ is displacement of base relative to the axis of reference,

and $x(t)$ is displacement of mass m relative to the base of the structure.

The number of dots refer to the order of derivatives with respect to time.

Therefore, the total displacement, $x_t(t)$, of the mass relative to the axis of reference is

$$x_t(t) = x_g(t) + x(t) \tag{2.2}$$

Rearrangement of equation (2.1) gives

$$\ddot{x}_g(t) = - [\ddot{x}(t) + 2\beta\omega\dot{x}(t) + \omega^2 x(t)] \tag{2.3}$$

wherein

β = fraction of critical damping = c/c_{cr}

c_{cr} = critical damping coefficient = $2m\omega$

ω = natural frequency of vibrations of the system = $\sqrt{(k/m)}$

Equation (2.3) represents the unknown ground excitation which is a function of time to be determined from the known quantities, i.e. $x(t)$ or $\ddot{x}(t)$, β and ω. It is to be noted that $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ are mathematically related.

(b) Multi degree-of-freedom system:

The matrix representation of the governing differential equations of motion for a n degrees-of-freedom system is given by

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = -\ddot{x}_g(t)\,[M]\{U\} \tag{2.4}$$

wherein

$[M]$ is a nxn mass matrix.

$[C]$ is a nxn matrix of damping coefficients.

$[K]$ is a nxn stiffness matrix.

$\{X(t)\}$ is a nx1 column vector with components equal to displacement of lumped masses relative to the base.

$x_g(t)$ is displacement of the base of the structure relative to the axis of reference.

$\{U\}$ is a nx1 column vector with all its components equal to unity.

In general, the mass and stiffness properties are distributed throughout the structure and it is therefore logical to use consistent mass and stiffness approach in formulating the mass and stiffness matrices[7]. However, as per the assumptions cited in the beginning of this chapter, the mass matrix turns out to be a nxn diagonal matrix.

The matrix representation, equation (2.4), consists of n-coupled second order differential equations. These can be uncoupled[8] through the modal matrix transformation which expresses the degrees of freedom in terms of the normal coordinates. The modal matrix is the matrix of the natural modes of the structurem which are obtained from the study of the undamped free vibration of the structure. The equations of motion for the free vibration of the structure are obtained from equation (2.4) by setting the matrix of damping coefficients as a null matrix and right hand side as a null vector, i.e.

$$[M]\{\ddot{X}(t)\} + [K]\{X(t)\} = \{0\} \qquad (2.5)$$

By definition, undamped free vibration is harmonic, therefore

$$\{X(t)\} = \{e\} \sin \omega t \qquad (2.6)$$

where

ω is frequency of vibration in a natufal mode.

$\{e\}$ is associated natural mode shape.

Substitution of equation (2.6) in equation (2.5) leads to an eigenvalue problem

$$\omega^2 [M]\{e\} = [K]\{e\} \qquad (2.7)$$

the solution to which leads to n natural frequencies, ω_i, and associated mode shapes, $\{e_i\}$.

The natural mode shapes are orthogonal with respect to each of the mass and stiffness matrices i.e.

$$\begin{aligned} &\{e_i\}^T [M] \{e_j\} = 0 \\ \text{and} \quad &\{e_i\}^T [K] \{e_j\} = 0 \end{aligned} \quad ; \quad i \neq j \qquad (2.8)$$

Let the natural mode shapes, $\{e_i\}$, be normalised such that

$$\{e_i\}^T [M]\{e_i\} = 1 \qquad (2.9)$$

and the displacement vector be expressed in terms of nx1 vector of normal coordinates, $\{q(t)\}$, by

$$\{X(t)\} = [Q]\{q(t)\} \qquad (2.10)$$

where $[Q]$ is a nxn modal matrix columns of which are the normalized mode shapes of the problem defined by equation (2.7).

Substitution of equation (2.10) into equation (2.4) gives

$$[M]\,[Q]\{\ddot{q}(t)\} + [C]\,[Q]\{\dot{q}(t)\} + [K]\,[Q]\{q(t)\}$$
$$= -\ddot{x}_g(t)\,[M]\{U\}$$

Premultiplication by $\{e_i\}^T$ gives

$$\{e_i\}^T [M] [Q]\{\ddot{q}(t)\} + \{e_i\}^T [C] [Q]\{\dot{q}(t)\} + \{e_i\}^T [K][Q]\{q(t)\}$$
$$= - \ddot{x}_g(t) \{e_i\}^T [M] \{U\}$$

Using equations (2.8) and (2.9) we get

$$\ddot{q}_i(t) + 2\beta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = - \alpha_i \ddot{x}_g(t)$$

or, $$\ddot{x}_g(t) = - \frac{1}{\alpha_i} [\ddot{q}_i(t) + 2\beta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t)] \quad (2.11)$$

where β_i = fraction of critical damping in the i^{th} mode,

and $\alpha_i = \{e_i\}^T [M] \{U\}$ is a scalar quantity called i^{th} modal participation factor. (2.12)

Equation (2.11) resembles equation (2.3) in form and prediction of $\ddot{x}_g(t)$ using equation (2.11) requires treatment identical to that for prediction of $\ddot{x}_g(t)$ using equation (2.3). Equation (2.11) requires information about $q_i(t)$ or $\ddot{q}_i(t)$. Information about $q_i(t)$ or $\ddot{q}_i(t)$ from $\{X(t)\}$ or $\{\ddot{X}(t)\}$ can be easily obtained by matrix operations on equation (2.10) which leads to

$$q_i(t) = \{e_i\}^T [M] \{X(t)\} \quad (2.13)$$

$$\ddot{q}_i(t) = \{e_i\}^T [M] \{\ddot{X}(t)\} \quad (2.14)$$

The presence of α_i, the i-th modal participation factor, in the denominator on the right hand side of equation (2.11) leads us to conclude that the errors in the prediction of $\ddot{x}_g(t)$ using equation (2.11) would be minimum if we pick up the largest value of α_i, $i = 1, 2, \ldots, n$. In other words, if α_j is the maximum of all α_i, $i = 1, 2, \ldots, n$ then it would be desirable to use equation (2.11) corresponding to j-th normal coordinate to predict the ground acceleration.

Thus in case of a multi degree-of-freedom system the prediction of $\ddot{x}_g(t)$ requires solution of only one equation similar to equation (2.3).

The set of equations given below are sufficient for prediction of $\ddot{x}_g(t)$ in case of a multi degree-of-freedom system:

$$\ddot{x}_g(t) = -\frac{1}{\alpha_j}\left[\ddot{q}_j(t) + 2\beta_j\,\omega_j\,\dot{q}_j(t) + \omega_j^2\,q_j(t)\right] \qquad (2.15)$$

where
$$q_j(t) = \{e_j\}^T [M] \{X(t)\}$$
$$\ddot{q}_j(t) = \{e_j\}^T [M] \{\ddot{X}(t)\}$$
$$\alpha_j = \{e_j\}^T [M] \{U\}$$

Suffix j corresponds to the predominant modal participation factor, α_j.

2.2 Choice of Excitation Parameter:

Any of the three mathematically related parameter-ground displacement, ground velocity and ground acceleration - can be considered to be the excitation parameter. In the present work, the ground displacement and ground acceleration have been considered to be the excitation parameters of interest due to the fact that most of the measuring instruments for vibration studies fall in either the class of accelerometers or displacement meters. This would thus facilitate the experimental verification of the results, if available.

2.3 Choice of Response Parameters:

Depending upon the physical properties of the measuring instruments mounted on structures, the response quantities fall either in class of displacements or accelerations. The quantities recorded may be either absolute or relative to ground; the nature of which depends upon the pickup element employed. In the present work, it has been assumed that quantities measured are relative to the ground movement and are recorded at the levels of lumped masses.

2.4 Development of Algorithms:

The algorithms are developed to predict the time history of excitation from the known response time history of a single degree-of-freedom system. But, as shown in article 2.2(b), the same algorithm can also be used to predict ground excitation for the multi degree-of-freedom systems.

2.4.1 DR-GD Algorithm:

This algorithm predicts the time history of ground displacement (GD) from the known time history of displacement response (DR). The governing equation for a single degree-of-freedom damped system is of the form

$$\ddot{x}_g(t) = -[\ddot{x}(t) + 2\beta\omega\dot{x}(t) + \omega^2 x(t)] \qquad (2.16)$$

wherein the different parameters are already defined. The integration of equation (2.16) with respect to t between 0 to t gives

$$\int_0^t \ddot{x}_g(t)dt = -[\dot{x}(t)-\dot{x}(0) + 2\beta\omega x(t) - 2\beta\omega x(0) + \omega^2 \int_0^t x(t)dt] \qquad \ldots (2.17)$$

Further integration and regrouping the terms yield

$$\int_0^t (\int_0^t \ddot{x}_g(t)dt)\, dt = -x(t) + (1+2\beta\omega t)x(0) + t\dot{x}(0) - 2\beta\omega\int_0^t x(t)dt - \omega^2 \int_0^t (\int_0^t x(t)dt)dt \qquad (2.18)$$

Numerical experimentation on equation (2.18) has shown that it is not possible to use this equation to advantage for prediction of $\ddot{x}_g(t)$. However, equation (2.18) can be very easily used for predicting $x_g(t)$ as follows:

$$\int_0^t \left(\int_0^t \ddot{x}_g(t)\,dt\right) dt = \int_0^t [\dot{x}_g(t) - \dot{x}_g(0)]\,dt$$

or,

$$\int_0^t \left(\int_0^t \ddot{x}_g(t)\,dt\right) dt = x_g(t) - x_g(0) - t\,\dot{x}_g(0) \qquad (2.19)$$

Thus,

$$x_g(t) = x_g(0) + t\,[\dot{x}_g(0) + \dot{x}(0)] - x(t) + (1+2\beta\omega t)\,x(0)$$
$$- 2\beta\omega \int_0^t x(t)\,dt - \omega^2 \int_0^t \left(\int_0^t x(t)\,dt\right) dt \qquad (2.20)$$

Denoting $x(t) = \ddot{z}(t)$, equation (2.20) can be rewritten as

$$x_g(t) = x_g(0) + t\,[\dot{x}_g(0) + \dot{x}(0)] - x(t) + (1+2\beta\omega t)\,x(0)$$
$$- 2\beta\omega\,\dot{z}(t) - \omega^2\,z(t) \qquad (2.21)$$

where $z(t)$ and $\dot{z}(t)$ are obtained from the solution of the differential equation

$$\ddot{z}(t) = x(t) \quad \text{with} \quad z(0) = \dot{z}(0) = 0 \qquad (2.22)$$

Any standard method for numerical solution of ordinary differential equation can be used to predict time history of ground displacement, $x_g(t)$. It is presumed that $x_g(0)$ is known, which is equal to zero for structures starting from rest. For such structures, $\dot{x}_g(0) + \dot{x}(0) = 0$.

The equations (2.21) and (2.22) constitute the set of basic equations for DR - GD algorithm.

2.4.2 DR-GA Algorithm:

This algorithm predicts the time history of ground acceleration (GA) from the known time history of displacement response.

For $t_i^* \leq t \leq t_{i+1}^*$, Fig. (2),

$$\ddot{x}(t) + 2\beta\omega\,\dot{x}(t) + \omega^2 x(t) = -[\ddot{x}_g(t_i) + b_i(t-t_i)] \qquad (2.23)$$

where the superscript * denotes the ends of a linear segment in the idealized ground acceleration plot, and,

b_i is the slope of the linear segment under consideration.

The solution of equation (2.23), for $t_i^* \leq t \leq t_{i+1}^*$, is

$$x(t) = e^{-\beta\omega(t-t_i)}\left[C_i \cos\omega\sqrt{(1-\beta^2)}(t-t_i) + D_i \sin\omega\sqrt{(1-\beta^2)}(t-t_i)\right] - \frac{1}{\omega^2}\left[\ddot{x}_g(t_i) + b_i\left(t-t_i - \frac{2\beta}{\omega}\right)\right] \qquad (2.24)$$

If it is assumed that

$t_i^* \leq t_i, t_{i+1}, t_{i+2}, t_{i+3} \leq t_{i+1}^*$, substitutions in equation (2.24) will give

$$x(t_i) = C_i - \frac{1}{\omega^2}\left[\ddot{x}_g(t_i) - \frac{2\beta}{\omega} b_i\right] \qquad (2.25)$$

$$x(t_{i+1}) = e^{-\beta\omega\Delta t_i}\left[C_i \cos\omega\sqrt{(1-\beta^2)}\,\Delta t_i + D_i \sin\omega\sqrt{(1-\beta^2)}\,\Delta t_i\right] - \frac{1}{\omega^2}\left[\ddot{x}_g(t_i) + b_i\left(\Delta t_i - \frac{2\beta}{\omega}\right)\right] \qquad (2.26)$$

$$x(t_{i+2}) = e^{-\beta\omega\Delta_2 t_i}\left[C_i \cos\omega\sqrt{(1-\beta^2)}\,\Delta_2 t_i + D_i \sin\omega\sqrt{(1-\beta^2)}\,\Delta_2 t_i\right] - \frac{1}{\omega^2}\left[\ddot{x}_g(t_i) + b_i\left(\Delta_2 t_i - \frac{2\beta}{\omega}\right)\right] \qquad (2.27)$$

$$x(t_{i+3}) = e^{-\beta\omega\Delta_3 t_i}\left[C_i \cos\omega\sqrt{(1-\beta^2)}\,\Delta_3 t_i + D_i \sin\omega\sqrt{(1-\beta^2)}\,\Delta_3 t_i\right] - \frac{1}{\omega^2}\left[x_g(t_i) + b_i\left(\Delta_3 t_i - \frac{2\beta}{\omega}\right)\right] \qquad (2.28)$$

wherein

$$\Delta t_i = t_{i+1} - t_i,\ \Delta_2 t_i = t_{i+2} - t_i,\ \Delta_3 t_i = t_{i+3} - t_i \quad (2.29)$$

Using the equations (2.25) to (2.28), it is possible to compute the values of the four unknowns, C_i, D_i, $\ddot{x}_g(t_i)$, b_i. This four point approach has been preferred over the step by step approach for prediction of ground acceleration. This is due to the fact that there is propagation of error in the latter approach as discussed in Appendix I. The approach followed here is free of such error propagations.

With such computed values of C_i, D_i, $\ddot{x}_g(t_i)$ and b_i, the values of displacement response at subsequent digitized time are computed and compared concurrently with the recorded values. An approximate range of a linear segment in the idealised ground acceleration time history is obtained by some tolerance for such comparison.

The identical set of equations are used to predict the approximate range of the next linear segment in the idealised ground acceleration time history. The point of intersection of the two linear segments gives the location of a point on the actual ground acceleration time history where a sharp change in slope is expected to occur. The same process is repeated along the entire time history of the known displacement response.

An observation of equations (2.25) to (2.28) suggests that the solution for the unknown parameters requires an inversion of a 3x3 matrix $[A]$, the elements of which are defined by

$$[A] = [A(\beta,\omega,t_i,\ t_{i+1},\ t_{i+2},\ t_{i+3})] = [a_{ij}];\ i,j = 1,2,3$$

$$a_{11} = e^{-\beta\omega(t_{i+1}-t_i)} \cos\omega\sqrt{(1-\beta^2)}(t_{i+1}-t_i) - e^{-\beta\omega(t_{i+3}-t_i)} \cos\omega\sqrt{(1-\beta^2)}(t_{i+3}-t_i)$$

$$a_{12} = e^{-\beta\omega(t_{i+1}-t_i)} \sin\omega\sqrt{(1-\beta^2)}(t_{i+1}-t_i) - e^{-\beta\omega(t_{i+3}-t_i)} \sin\omega\sqrt{(1-\beta^2)}(t_{i+3}-t_i)$$

$$\begin{aligned}
a_{13} &= -\frac{1}{\omega^2}(t_{i+1} - t_{i+3}) \\
a_{21} &= -1 + e^{-\beta\omega(t_{i+3}-t_i)} \cos\omega\sqrt{(1-\beta^2)}\,(t_{i+3}-t_i) \\
a_{22} &= e^{-\beta\omega(t_{i+3}-t_i)} \sin\omega\sqrt{(1-\beta^2)}(t_{i+3}-t_i) \\
a_{23} &= -\frac{1}{\omega^2}(t_{i+3}-t_i) \\
a_{31} &= -1 + e^{-\beta\omega(t_{i+2}-t_i)} \cos\omega\sqrt{(1-\beta^2)}(t_{i+2}-t_i) \\
a_{32} &= e^{-\beta\omega(t_{i+2}-t_i)} \sin\omega\sqrt{(1-\beta^2)}\,(t_{i+2}-t_i) \\
a_{33} &= -\frac{1}{\omega^2}(t_{i+2} - t_i)
\end{aligned} \tag{2.30}$$

In general, the value of the above elements will vary according to the step size of the digitization, and thus computation of matrix $[A]$ will be necessary for each step of digitization. However, in case of equally spaced digitized time coordinates, the matrix $[A]$ remains constant althrough, and is required to be computed only once.

Thus, we have

$$[A] \begin{Bmatrix} C_i \\ D_i \\ b_i \end{Bmatrix} = \begin{Bmatrix} x(t_{i+1}) - x(t_{i+3}) \\ x(t_{i+3}) - x(t_i) \\ x(t_{i+2}) - x(t_i) \end{Bmatrix} \tag{2.31}$$

$$\ddot{x}_g(t_i) = \omega^2 [C_i - x(t_i)] + \frac{2\beta}{\omega} b_i \qquad (2.32)$$

and, $$\ddot{x}_g(t) = \ddot{x}_g(t_i) + b_i(t-t_i); \quad t_i^* \leq t \leq t_{i+1}^* \qquad (2.33)$$

The equations (2.30) to (2.33) constitute set of basic equations for DR-GA algorithm.

2.4.3 AR-GA Algorithm:

This algorithm predicts the time history of ground acceleration fwom the knowledge of acceleration response (AR) of structures.

Differentiating equation (2.23) twice with respect to time, we get,

$$x^{iv}(t) + 2\beta\omega \dddot{x}(t) + \omega^2 \ddot{x}(t) = 0 \qquad (2.34)$$

Equation (2.34) is valid for $t_i^* \leq t \leq t_{i+1}^*$. Solution of Equation (2.34) gives

$$\ddot{x}(t) = e^{-\beta\omega(t-t_i^*)} \left[C_i \cos \omega\sqrt{(1-\beta^2)}\,(t-t_i^*) + D_i \sin \omega\sqrt{(1-\beta^2)}\,(t-t_i^*)\right] \qquad (2.35)$$

wherein

$$C_i = \ddot{x}(t_i^*)$$

$$D_i = \frac{\dfrac{\ddot{x}(t_{i+1}^*)}{e^{-\beta\omega(t_{i+1}^*-t_i^*)}} - \ddot{x}(t_i^*) \cos \omega\sqrt{(1-\beta^2)}\,(t_{i+1}^* - t_i^*)}{\sin \omega\sqrt{(1-\beta^2)}\,(t_{i+1}^* - t_i^*)} \qquad (2.36)$$

Integrations of equation (2.35) result in the following expressions for $\dot{x}(t)$ and $x(t)$;

$$\dot{x}(t) = \dot{x}(t_i^*) + C_i\ I_1\ (\beta, \omega, t, t_i^*) + D_i\ I_2\ (\beta, \omega, t, t_i^*) \quad \ldots \quad (2.37)$$

$$x(t) = x(t_i^*) + \left[\dot{x}(t_i^*) + \frac{C_i\beta + D_i\sqrt{(1-\beta^2)}}{\omega}\right](t-t_i^*) - \frac{C_i\beta + D_i\sqrt{(1-\beta^2)}}{\omega}\ I_1\ (\beta, \omega, t, t_i^*) + \frac{C_i\sqrt{(1-\beta^2)} - D_i\beta}{\omega}\ I_2\ (\beta, \omega, t, t_i^*) \quad (2.38)$$

where

$$I_1(\beta, \omega, t, t_i^*) = \frac{1}{\omega}\left[\beta\left\{1 - e^{-\beta\omega(t-t_i^*)}\cos\omega\sqrt{(1-\beta^2)}(t-t_i^*)\right\} + \sqrt{(1-\beta^2)}\ e^{-\beta\omega(t-t_i^*)}\sin\omega\sqrt{(1-\beta^2)}(t-t_i^*)\right] \quad \ldots \quad (2.39)$$

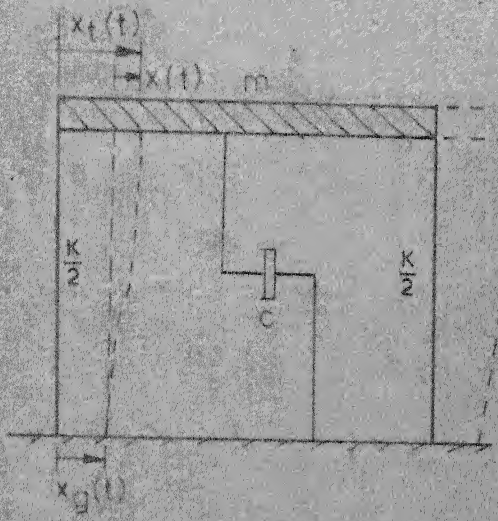
$$I_2(\beta, \omega, t, t_i^*) = \frac{1}{\omega}\left[-\beta\ e^{-\beta\omega(t-t_i^*)}\sin\omega\sqrt{(1-\beta^2)}\ (t-t_i^*) + \sqrt{(1-\beta^2)}\left\{1 - e^{-\beta\omega(t-t_i^*)}\cos\omega\sqrt{(1-\beta^2)}(t-t_i^*)\right\}\right] \quad \ldots \quad (2.40)$$

Use of equations (2.3), and (2.37) to (2.40) gives

$$\ddot{x}_g(t_{i+1}^*) = -\left\{\ddot{x}(t_{i+1}^*) + 2\beta\omega\dot{x}(t_{i+1}^*) + \omega^2\ x(t_{i+1}^*)\right\} \quad (2.41)$$

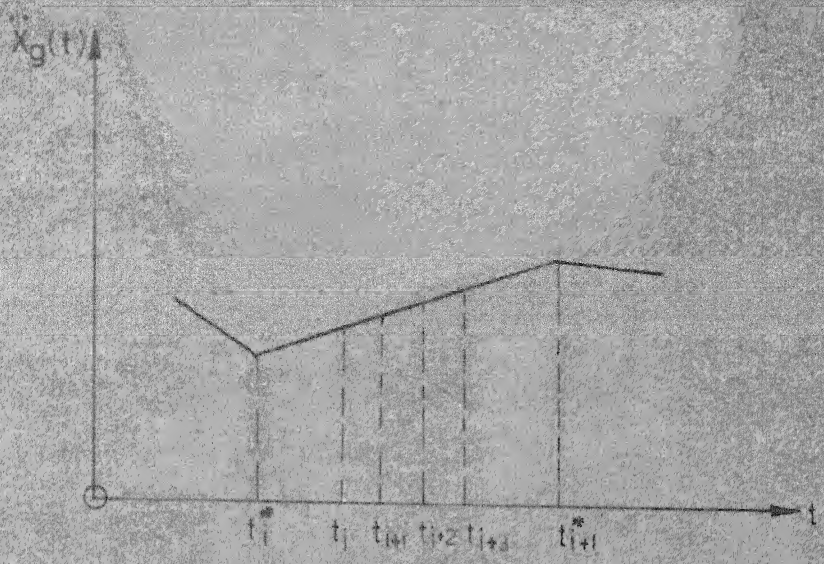
Recursive use of equations (2.41) alongwith equations (2.37) to (2.40) gives the desired information about the time history of ground acceleration. In order to initiate the algorithm, the initial conditions of the structure i.e. $x(0)$ and $\dot{x}(0)$ are presumed to be known.

Ground displacement and velocity, if required, can be computed from the time versus ground acceleration plot using the algorithm given by Berg and Housner[9] which is given in Appendix II in this report.

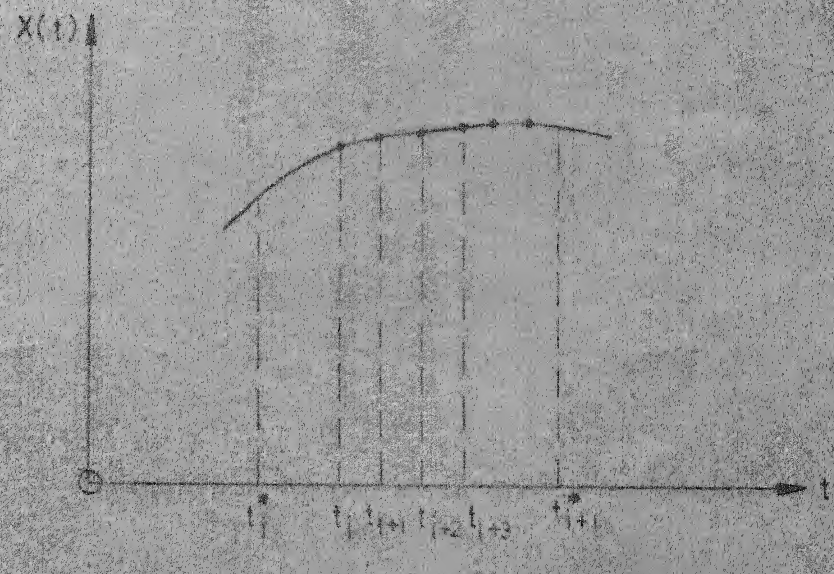


GROUND MOTION EXCITATION OF A SINGLE D.O.F SYSTEM

FIGURE 1



(i) IDEALIZED GROUND ACCELERATION PLOT



(ii) KNOWN DISPLACEMENT RESPONSE PLOT

FIGURE 2

CHAPTER 3

ILLUSTRATIVE EXAMPLES

The algorithms, discussed in chapter 2, have been applied to one and two degree-of-freedom systems. Results obtained show good agreement between the actual and computed ground acceleration versus time plots. Both the slow ground motion and strong ground motion excitations have been predicted successfully. The distinction between the slow ground motion and the strong ground motion is a qualitative one. It depends upon the number of zero crossings in the time history. The larger the number of zero crossings the stronger is the motion.

In order to check the algorithms discussed, displacement response of the selected one and two degree-of-freedom structures subjected to known ground acceleration time history are obtained, appendix III. These digitized displacement response along with physical characteristics of structure, in turn, were fed as the input for the algorithm discussed. The output corresponds to the desired ground excitation. The examples chosen are single and double storied planar shear building structures.

For all the examples presented herein, (i) IBM 7044 digital computer was used during computations and (ii) following assumptions were made.

(a) Total mass of the structure is lumped at the level of floors and roof.

(b) The flexural rigidity of the girder is large in comparison to that of the columns.

(c) The deformations in the members due to axial forces are negligible.

(d) The vertical translation and the rotation about the horizontal axes are negligible.

(e) The system starts from rest.

Single degree-of-freedom structure:

Example 1:

Consider the undamped single degree-of-freedom structure shown in Fig. 3. The displacement response of the structure at the level of the lumped mass m has been taken to be known, Fig.4, and DR-GA algorithm has been used to predict the ground acceleration. An absolute tolerence of 0.001 cm is taken to predict the approximate range of a linear segment in the idealized time history of ground acceleration.

In this example, the physical characteristics are:

Natural frequency = 6.0 radians per second.

Damping as a fraction
of critical damping = 0.0

The digitized values of the displacement response, obtained as punched output using method described in appendix III, shown in Fig. 4 were fed as input for the DR-GA algorithm. The points obtained on the predicted plot of ground acceleration versus time are shown in Fig. 5. The problem was rerun, this time with the digitized values of displacement response being given only upto four places of decimal. The results obtained from this run are also shown in Fig. 5.

The points as predicted by the algorithm show good agreement with the actual plot.

<u>Example 2</u>:

The physical system of example 1, Fig. 3, was again considered but this time response parameter, under consideration, was the acceleration, $\ddot{x}(t)$, of the mass, m, relative to ground, Fig. 6. The AR-GA algorithm was used to predict the ground acceleration time history. Actual plot and the computed plot of the ground accelerations are shown in Fig. 7. These show excellent results.

<u>Example 3</u>:

This example once again considers the physical system, Fig. 3, as discussed in the previous examples but the acceleration response plot, as shown in Fig. 8, is the response of the structure subjected to a somewhat arbitrary strong motion ground acceleration, which is almost periodic in nature, Fig. 9. The ground accelerations have been computed using the AR-GA algorithm. The points on the ground acceleration versus time plot, as computed by this algorithm, and the actual plot are almost coincident, shown in Fig. 9.

<u>Two Degrees-of-freedom structure</u>:

The remaining illustrations deal with two storied framed structure, shown in Fig. 10. The displacement response plots as considered correspond to N69W component of the ground acceleration recorded at Taft, California on July 21, 1952. The DR-GA algorithm is used to reproduce this shock. A relative tolerance of 10^{-6} is set to predict the approximate range of a linear segment in the idealized time history of ground acceleration.

As shown in Fig. 10, the masses lumped at the two floors are

$$\begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 1.0 \end{Bmatrix} \quad \text{F-Sec}^2/\text{cm} \qquad (3.1)$$

where F is the unit of force.

The stiffness of the two stories are

$$\begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = \begin{Bmatrix} 75.0 \\ 50.0 \end{Bmatrix} \quad \text{F/cm} \qquad (3.2)$$

The 2x2 stiffness matrix and the 2x2 diagonal mass matrix turn out to be

$$[K] = \begin{bmatrix} 125.0 & -50.0 \\ -50.0 & 50.0 \end{bmatrix} \quad \text{F/cm} \qquad (3.3)$$

$$[M] = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \quad \text{F-Sec}^2/\text{cm} \qquad (3.4)$$

The fundamental natural frequency, ω_1, and the associated mode shape, $\{e_1\}$, are given by

$$\omega_1 = 5.0 \text{ radians per second}$$

and

$$\{e_1\} = \begin{Bmatrix} 0.44721361 \\ 0.89442717 \end{Bmatrix} \qquad (3.5)$$

where the mode shape, $\{e_1\}$, is normalised as

$$\{e_1\}^T [M] \{e_1\} = 1.0$$

For this structure the first modal participation factor, α_1, is the largest.

The value of the gravitational constant, g, is taken to be 980.0 cm/sec^2.

Through a simple illustration given below it is shown that the DR-GA algorithm fares better compared to central finite difference method. Equal time intervals have been used between succeeding digitized time coordinates in order to compare the likelihood of errors in the computed values of ground acceleration. This comparison is made for undamped system with a natural frequency of 5.0 radians per second.

Using the central finite difference approximation for derivatives[10]

$$\ddot{x}(t_i) = \frac{x(t_{i+1}) - 2x(t_i) + x(t_{i-1})}{h^2} \qquad (3.6)$$

where $h = t_i - t_{i-1} = t_{i+1} - t_i$ (3.7)

Therefore, from equation (2.3) we have

$$\ddot{x}_g(t_i) = -\left[\frac{x(t_{i+1}) + x(t_{i-1})}{h^2} + \left(\omega^2 - \frac{2}{h^2}\right) x(t_i)\right] \qquad (3.8)$$

If ϵ is the magnitude of the maximum error incurred in digitization of any value of x, which can be either way, the magnitude of the maximum error, $\epsilon[\ddot{x}_g(t_i)]$, in $\ddot{x}_g(t_i)$ is given by

$$\epsilon[\ddot{x}_g(t_i)] = \left(\frac{4}{h^2} - \omega^2\right) \epsilon \qquad (3.9)$$

In table 1, expressions for C_i and $\ddot{x}_g(t_i)$, obtained by using DR-GA algorithm, are given for h = 0.01, 0.02, 0.03, 0.04 and 0.05 second. In table 2, the magnitude of the maximum errors likely to occur in the resulting computation of $\ddot{x}_g(t_i)$ using the DR-GA algorithm and finite difference method are listed to show how the DR-GA algorithm fares as compared to well known central finite difference method.

Table 1

h Sec.	C_i	$\ddot{x}_g(t_i)$
0.01	$-31.2\ x(t_i)+78.4\ x(t_{i+1})$ $-63.3\ x(t_{i+2})+16.1\ x(t_{i+3})$	$25[-32.2\ x(t_i)+78.4\ x(t_{i+1})$ $-63.3\ x(t_{i+2})+16.1\ x(t_{i+3})]$
0.02	$-\ 7.2\ x(t_i)+18.4\ x(t_{i+1})$ $-15.3\ x(t_{i+2})+4.1\ x(t_{i+3})$	$25[-\ 8.2\ x(t_i)+18.4\ x(t_{i+1})$ $-15.3\ x(t_{i+2})+4.1\ x(t_{i+3})]$
0.03	$-\ 2.73\ x(t_i)+7.32\ x(t_{i+1})$ $-\ 6.45\ x(t_{i+2})+1.86\ x(t_{i+3})$	$25[-\ 3.73\ x(t_i)+7.32\ x(t_{i+1})$ $-\ 6.45\ x(t_{i+2})+1.86\ x(t_{i+3})]$
0.04	$-\ 1.18\ x(t_i)+3.44\ x(t_{i+1})$ $-\ 3.35\ x(t_{i+2})+1.09\ x(t_{i+3})$	$25[-\ 2.18\ x(t_i)+3.44\ x(t_{i+1})$ $-\ 3.35\ x(t_{i+2})+1.09\ x(t_{i+3})]$
0.05	$-\ 0.46\ x(t_i)+1.65\ x(t_{i+1})$ $-\ 1.92\ x(t_{i+2})+0.73\ x(t_{i+3})$	$25[-\ 1.46\ x(t_i)+1.65\ x(t_{i+1})$ $-\ 1.92\ x(t_{i+2})+0.73\ x(t_{i+3})]$

Example 4:

Displacement response of the structure, Fig. 10, subjected to the ground motion recorded at Taft, are shown in Figs. 11 to 13.

<u>Table 2</u>

h Sec.	Magnitude of maximum error likely in computation of $\ddot{x}_g(t_i)$ by	
	The developed algorithm DR-GA	Central finite difference method
0.01	4750 ϵ	39975 ϵ
0.02	1150 ϵ	9975 ϵ
0.03	484 ϵ	4419 ϵ
0.04	251.5 ϵ	2475 ϵ
0.05	144 ϵ	1575 ϵ

Figs. 11 and 12 show the displacement response with zero percent damping in the two modes whereas Fig. 13 shows the displacement response of the structure with one percent damping in the two modes. The latter plot corresponds to only first six seconds of the record at Taft. The corresponding ground acceleration plots are respectively shown in Figs. 14 and 15. They compare well with actual ground acceleration plots.

<u>Example 5</u>:

The structure, Fig. 10, has been assumed to have two and a half percent damping in both the modes and the displacement response of the structure at the levels of lumped masses were computed. The response plot is shown in Fig. 16. However, during the prediction of ground acceleration corresponding to this response using DR-GA algorithm, the structure was assumed to be undamped in the fundamental mode. The results, as shown in Fig. 17, point out that even in the absence of knowledge of the exact value of the fraction of critical damping in the fundemental mode, the DR-GA algorithm may be expected to give satisfactory results.

Example 6:

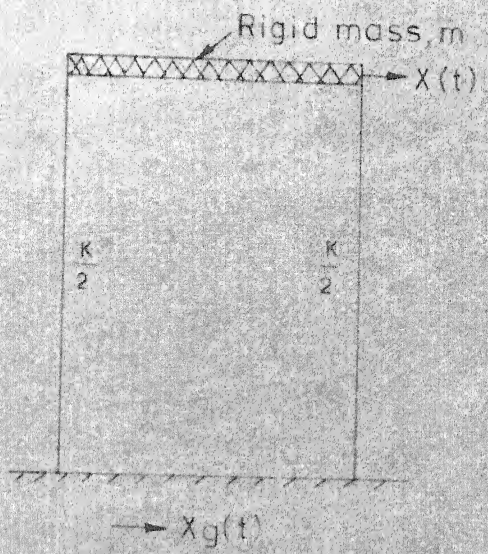
This illustrative example is considered to study the sensitivity of the algorithm when the displacement response of the masses are perturbed. This study is particularly useful because of the limitations of the displacement meter to give precise informations during measurements. The actual and perturbed values of displacement response for two degrees-of-freedom undamped system, Fig. 10, are shown in Table 3.

Table 3

Coordinates of the actual response plot			Coordinates of the perturbed plot		
t Sec.	$x_1(t)$ cm	$x_2(t)$ cm	t Sec.	$x_1(t)$ cm	$x_2(t)$ cm
0.0100	$0.30864228*10^{-3}$	$0.37939300*10^{-3}$	0.0100	$0.20*10^{-3}$	$0.30*10^{-3}$
0.0200	$0.10678828*10^{-2}$	$0.12465664*10^{-2}$	0.0240	$0.10*10^{-2}$	$0.13*10^{-2}$
0.0300	$0.19432205*10^{-2}$	$0.22361298*10^{-2}$	0.0320	$0.19*10^{-2}$	$0.22*10^{-2}$
0.0400	$0.25757941*10^{-2}$	$0.29944099*10^{-2}$	0.0440	$0.25*10^{-2}$	$0.29*10^{-2}$
0.0500	$0.27528822*10^{-2}$	$0.33120033*10^{-2}$	0.0520	$0.27*10^{-2}$	$0.33*10^{-2}$

Other values of $x_1(t)$ and $x_2(t)$ were considered upto four places of decimal only keeping time coordinate as on the actual plot.

The computed plot, using DR-GA algorithm, for the first one second of the record is given in Fig. 18. The computed and the actual plots show that the algorithm is locally sensitive to the perturbations in the displacement response. More the relative perturbation in the displacement response, more is observed to be the difference in the computed plots and actual plot in the local region of perturbation.



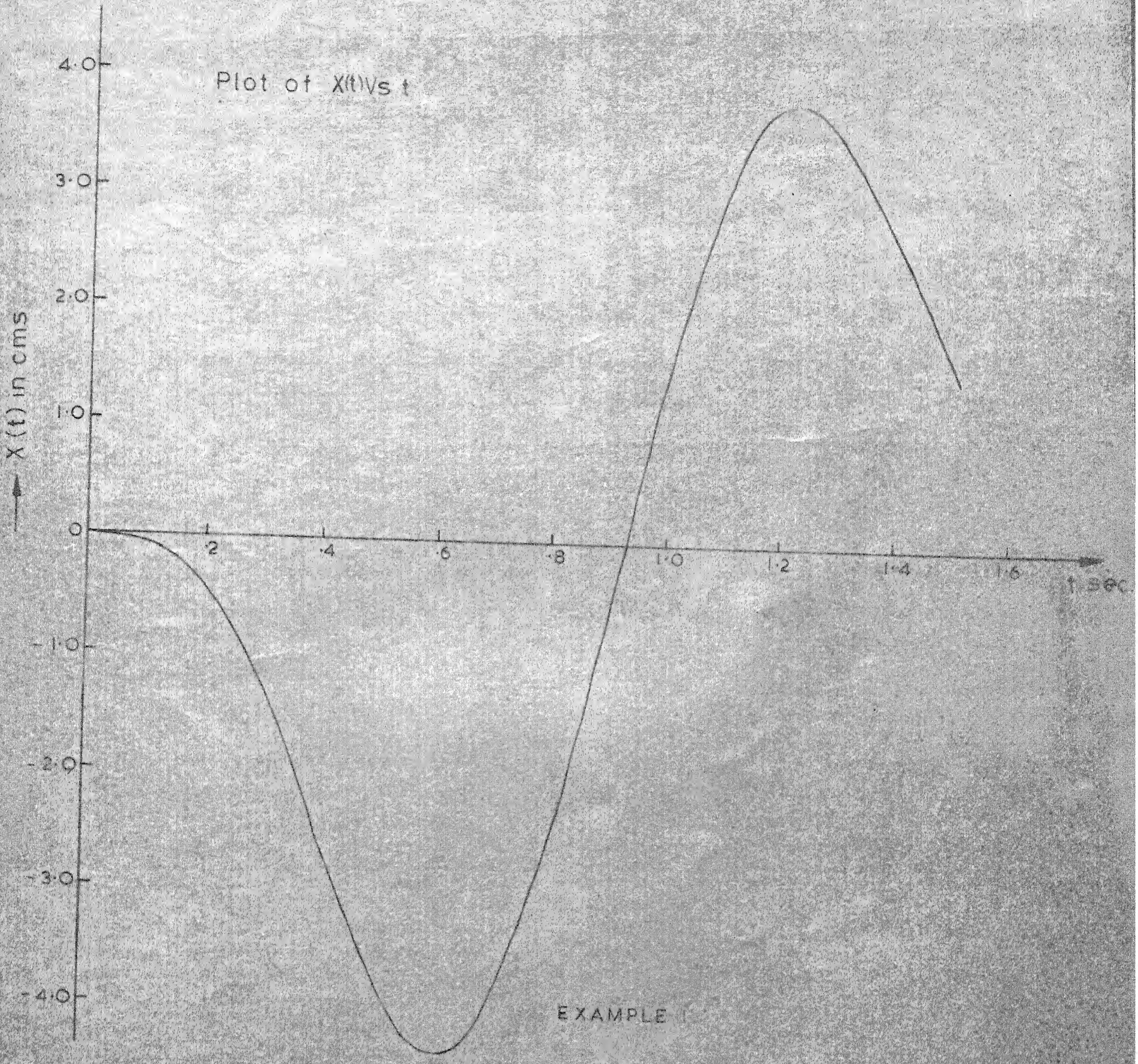
TOTAL STIFFNESS OF THE COLUMNS = K

NATURAL FREQUENCY OF THE STRUCTURE,
ω = 6·0 RADIANS PER SECOND

EXAMPLES 1,2,3

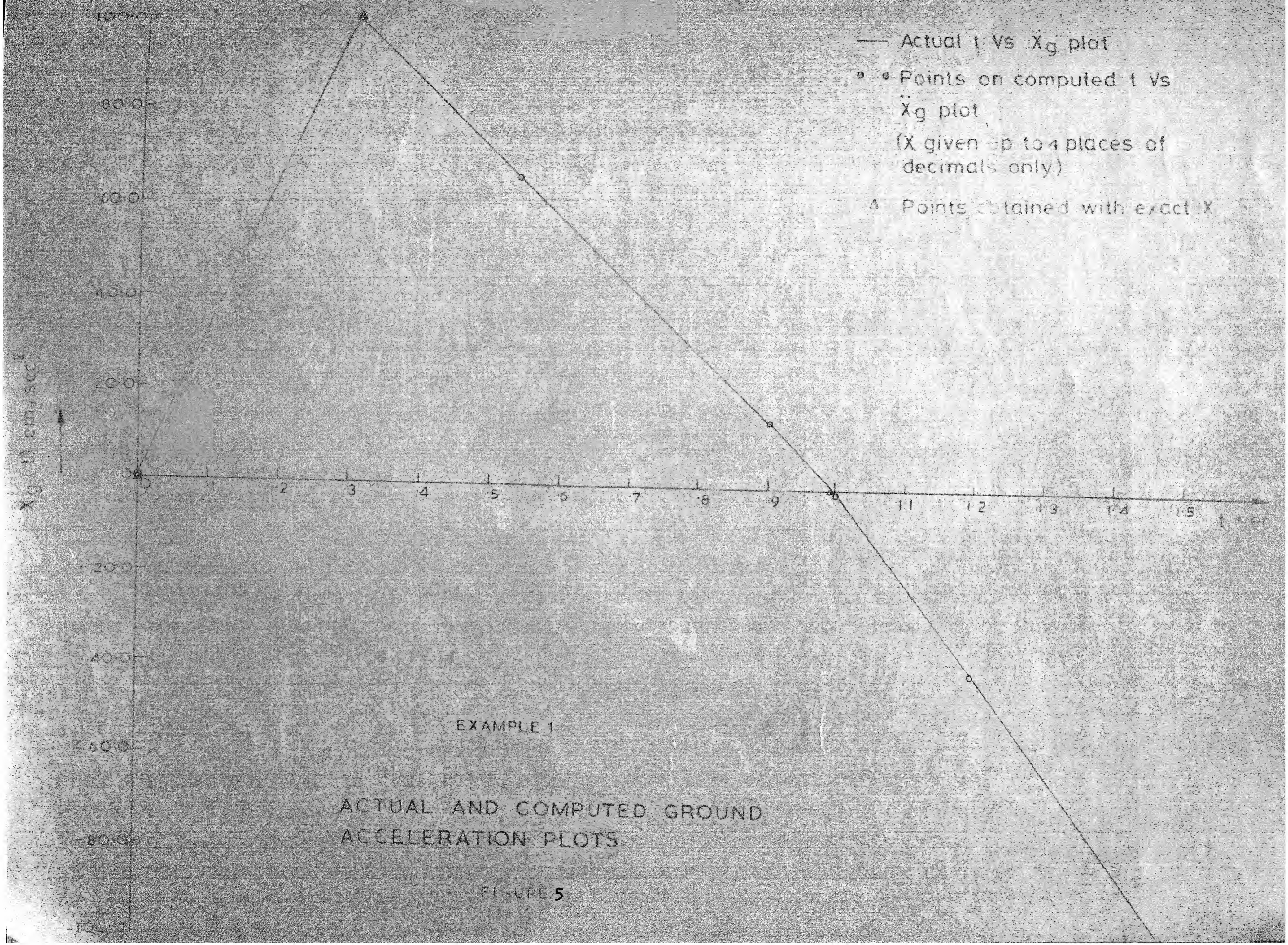
UNDAMPED SINGLE DEGREE OF FREEDOM STRUCTURE

FIGURE 3



PLOT OF DISPLACEMENT RESPONSE Vs TIME

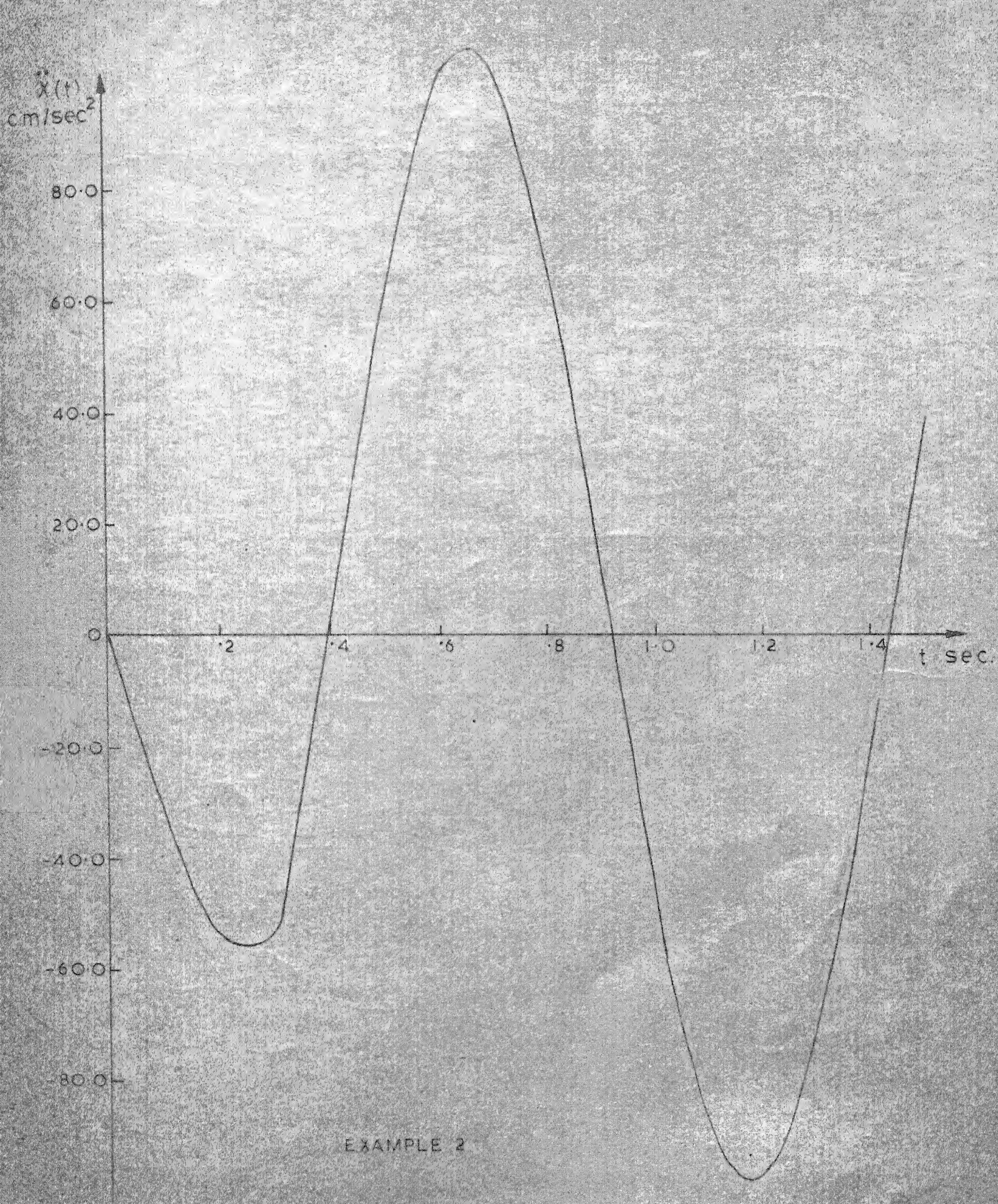
FIGURE 4



EXAMPLE 1

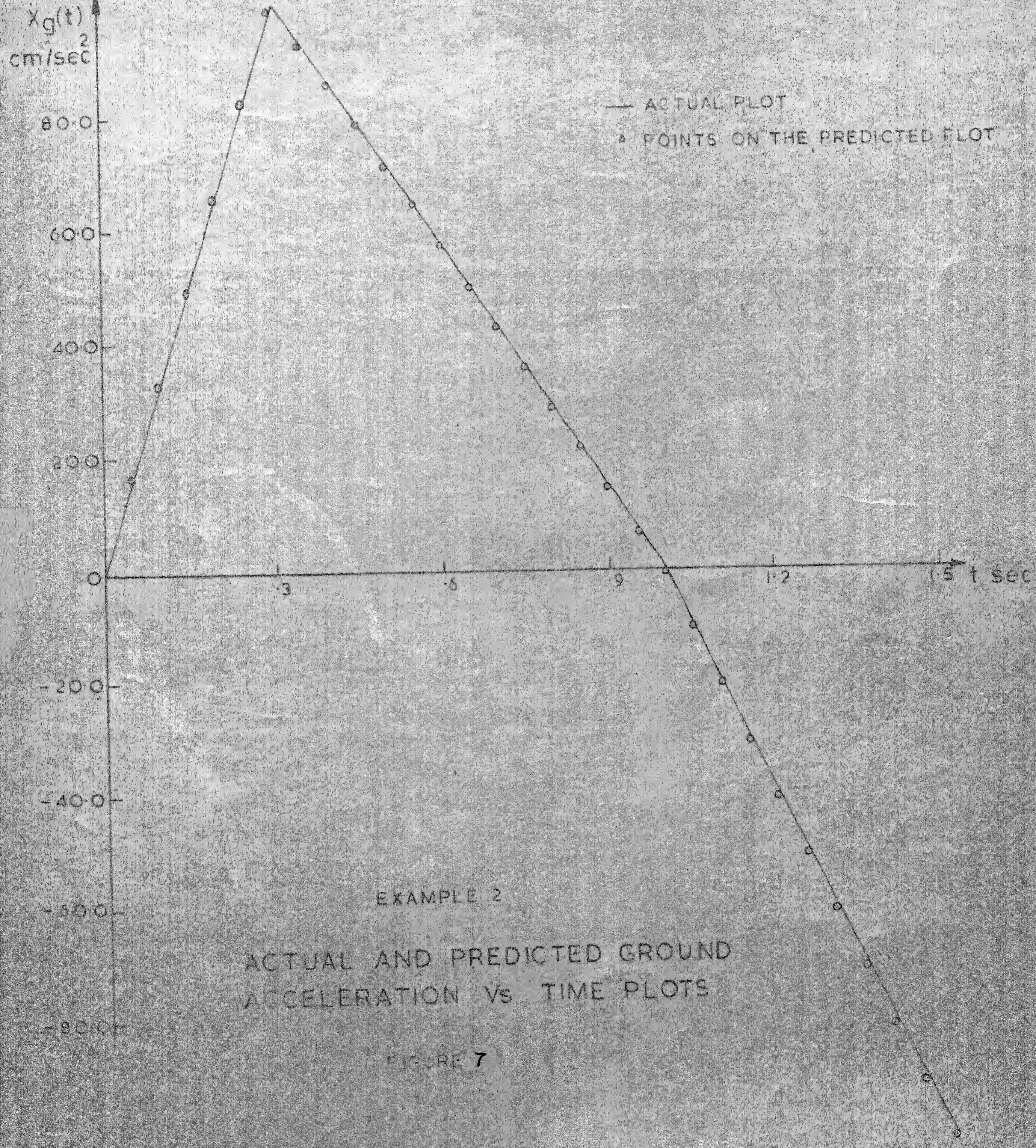
ACTUAL AND COMPUTED GROUND ACCELERATION PLOTS

FIGURE 5



ACCELERATION RESPONSE PLOT

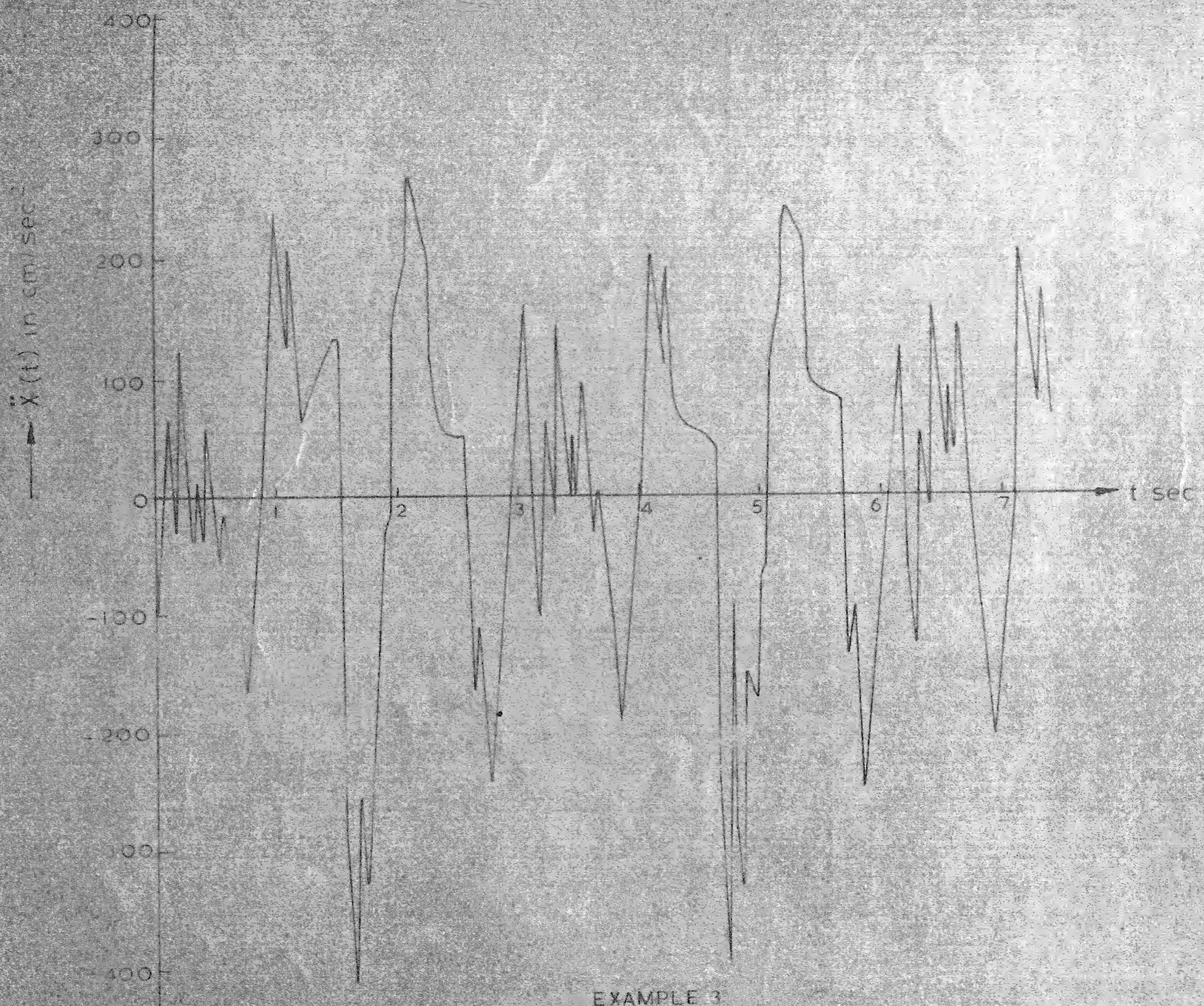
FIGURE 6



EXAMPLE 2

ACTUAL AND PREDICTED GROUND ACCELERATION Vs TIME PLOTS

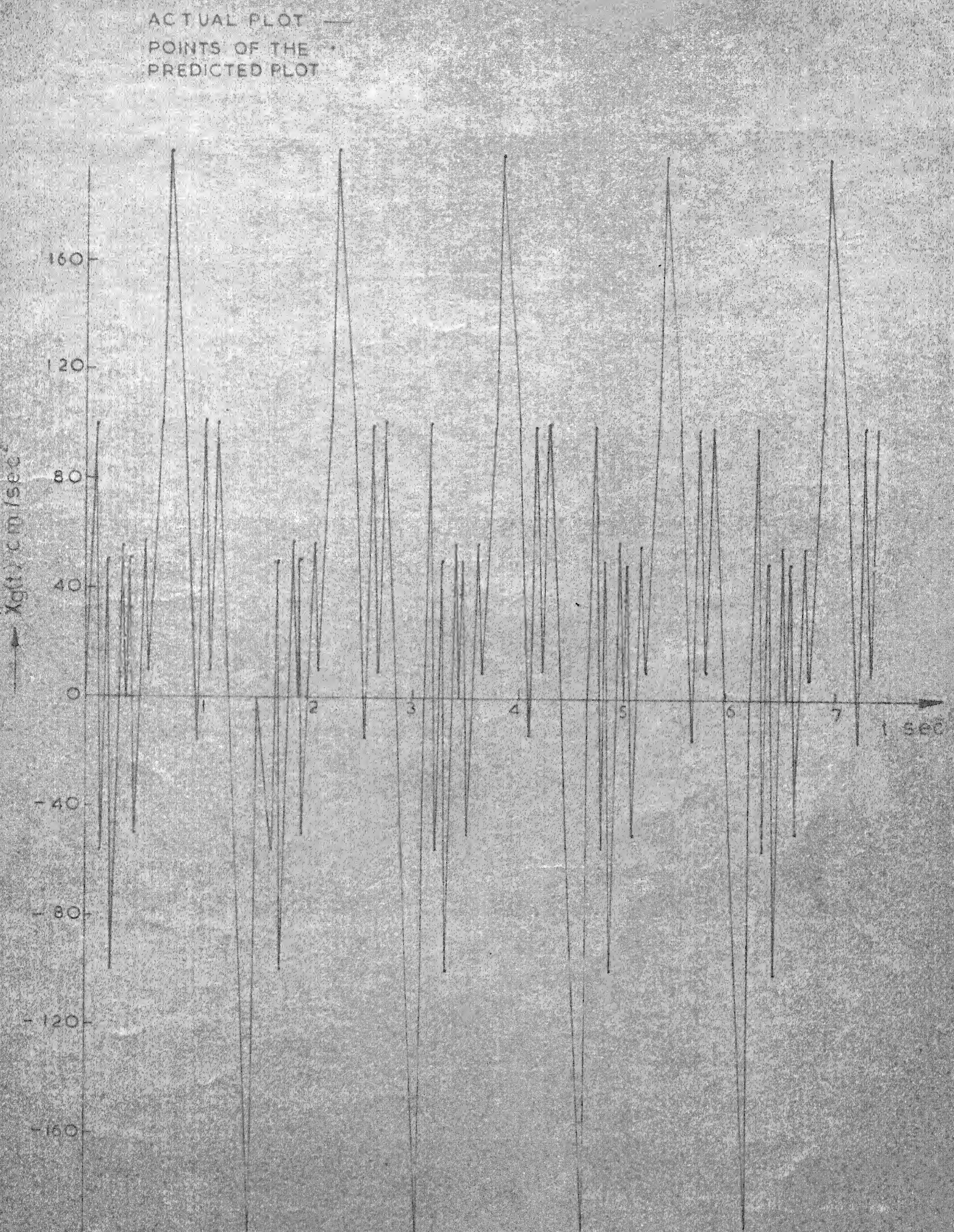
FIGURE 7



EXAMPLE 3

ACCELERATION RESPONSE Vs TIME PLOT

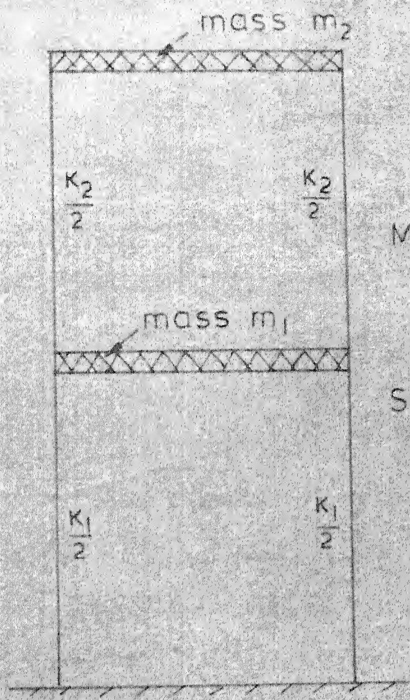
FIGURE 8



EXAMPLE 3

GROUND ACCELERATION Vs TIME PLOT

FIGURE 9



Masses are

$$\begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 1.0 \end{Bmatrix} \text{F-sec}^2/\text{cm}$$

Stiffness of different stories are

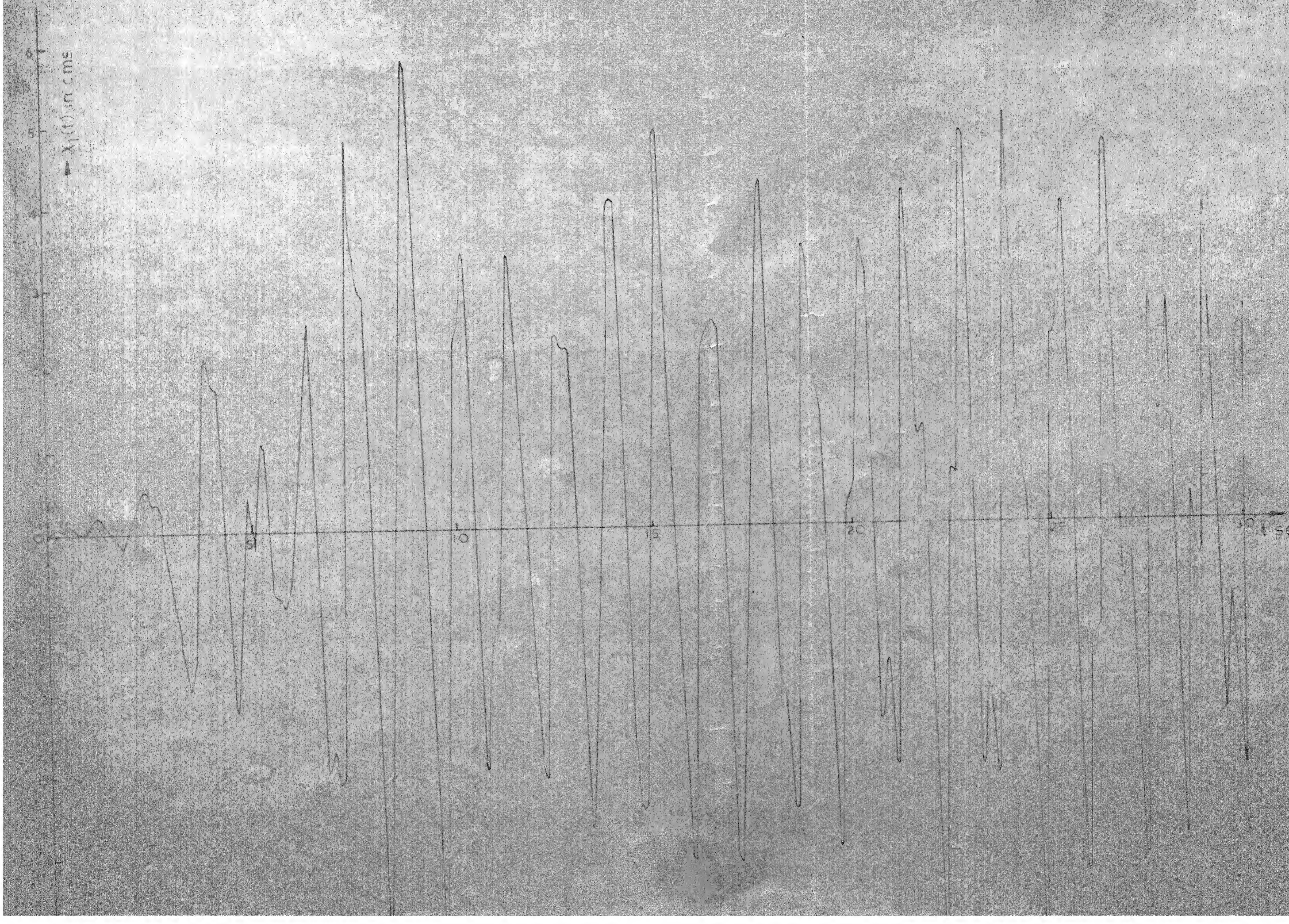
$$\begin{Bmatrix} K_1 \\ K_2 \end{Bmatrix} = \begin{Bmatrix} 75.0 \\ 50.0 \end{Bmatrix} \text{F/cm}$$

F : Unit of force

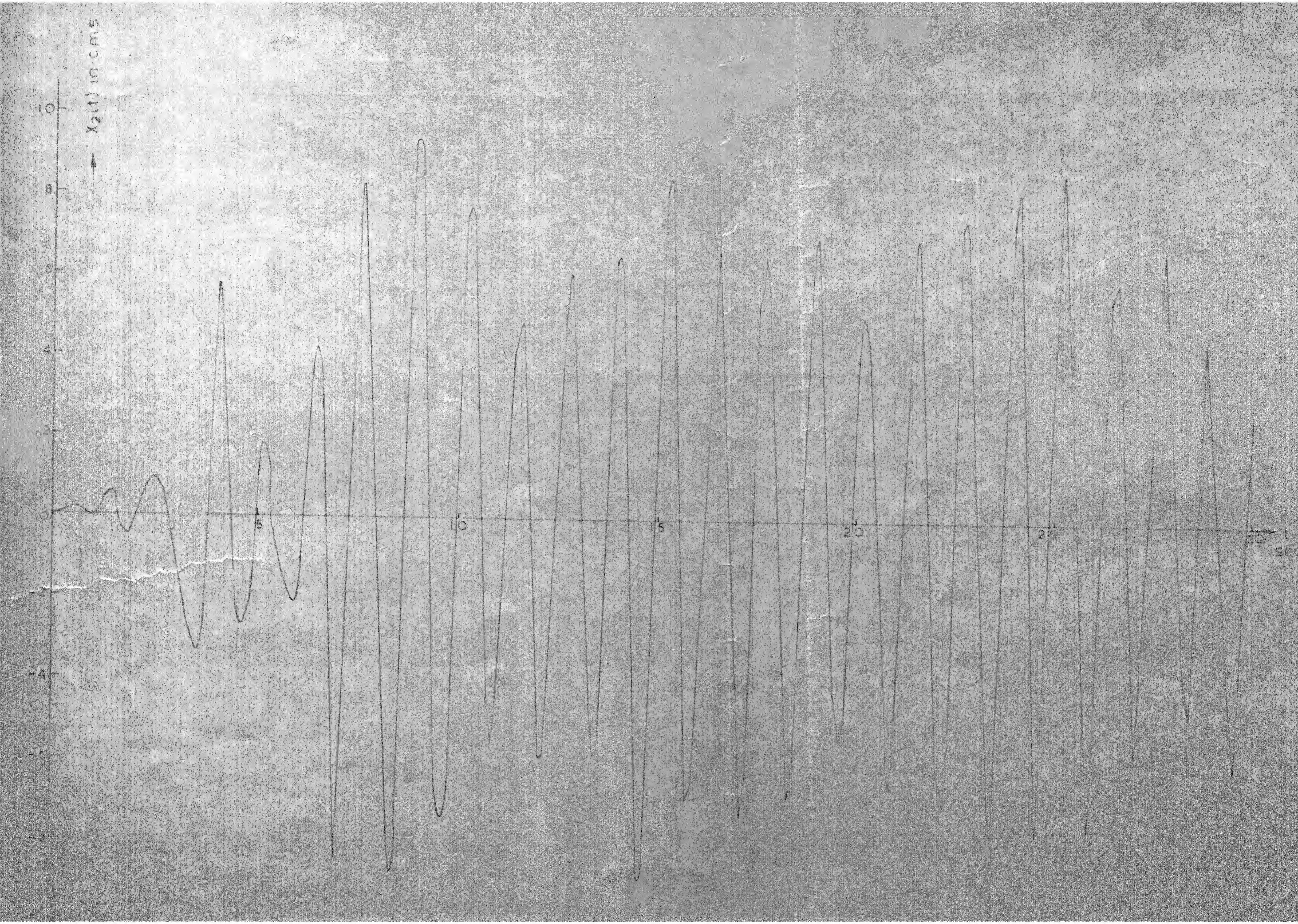
EXAMPLE 4,3,5

TWO STORIED FRAMED STRUCTURE

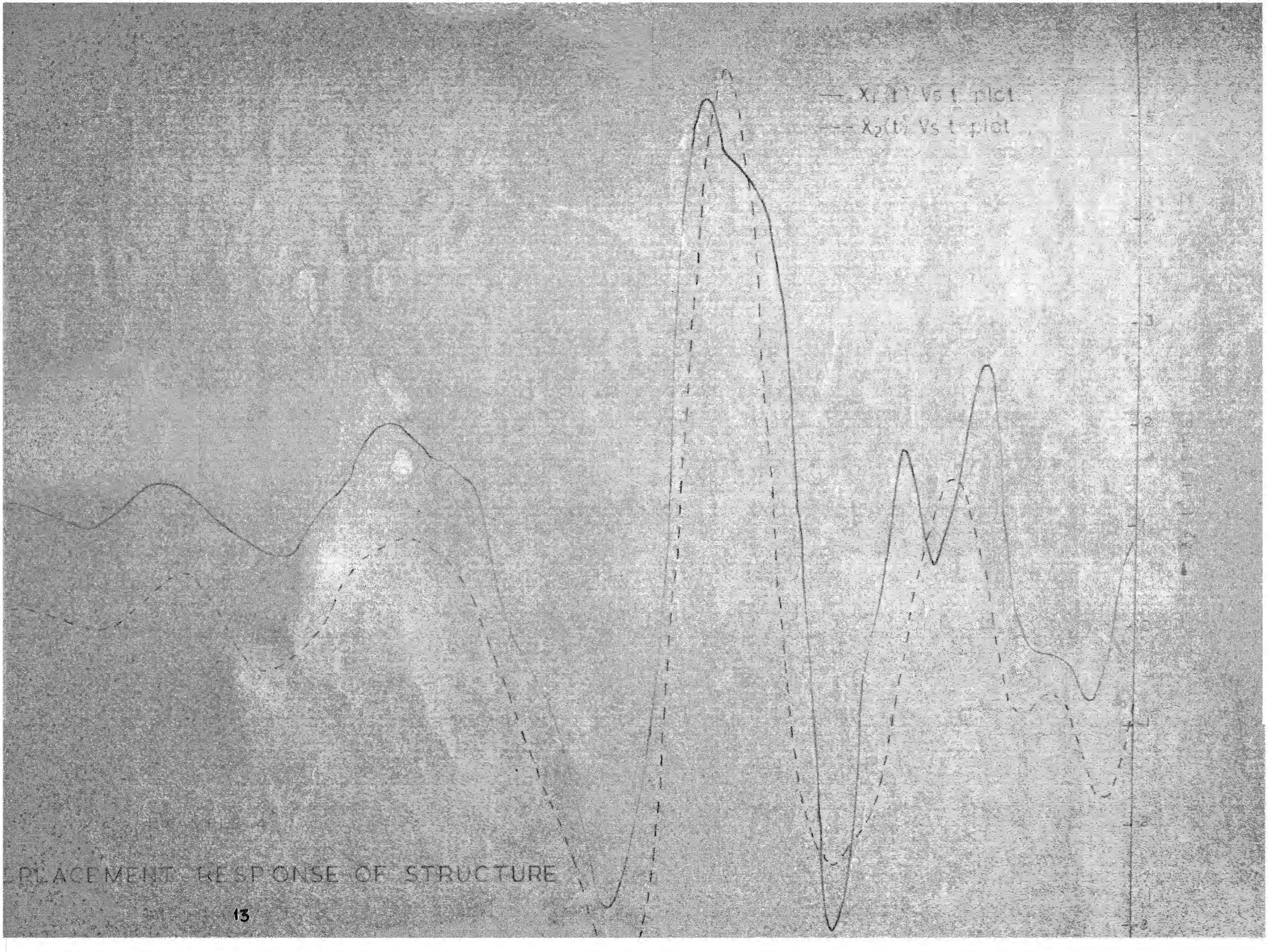
FIGURE 10



$X_1(t)$ in cms
t sec
0
5
10
15
20
25
30
1
2
3
4
5
6

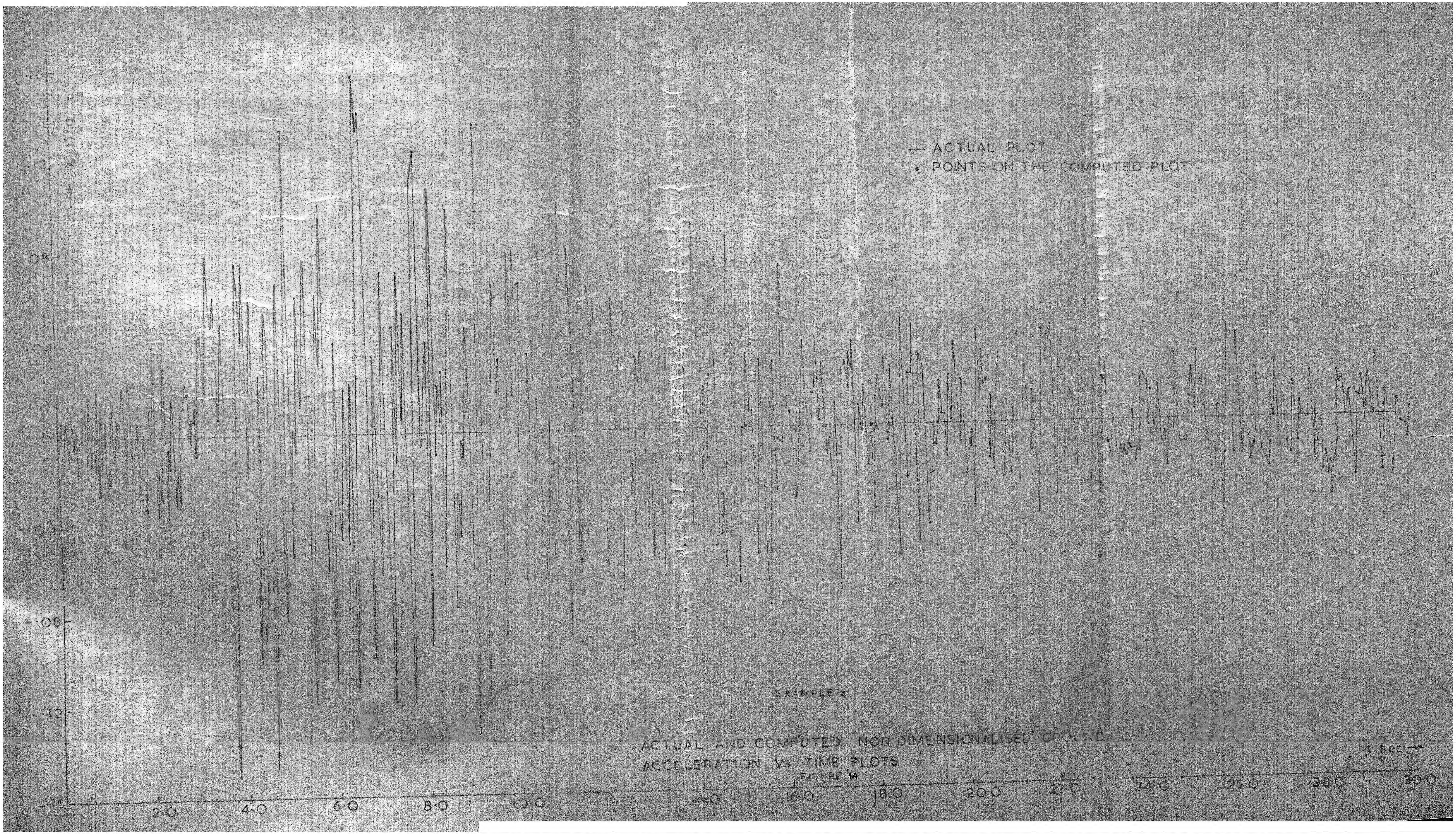


$X_2(t)$ in cms
10
8
6
4
2
0
-2
-4
-6
-8
5
10
15
20
25
30
t
sec



PLACEMENT RESPONSE OF STRUCTURE

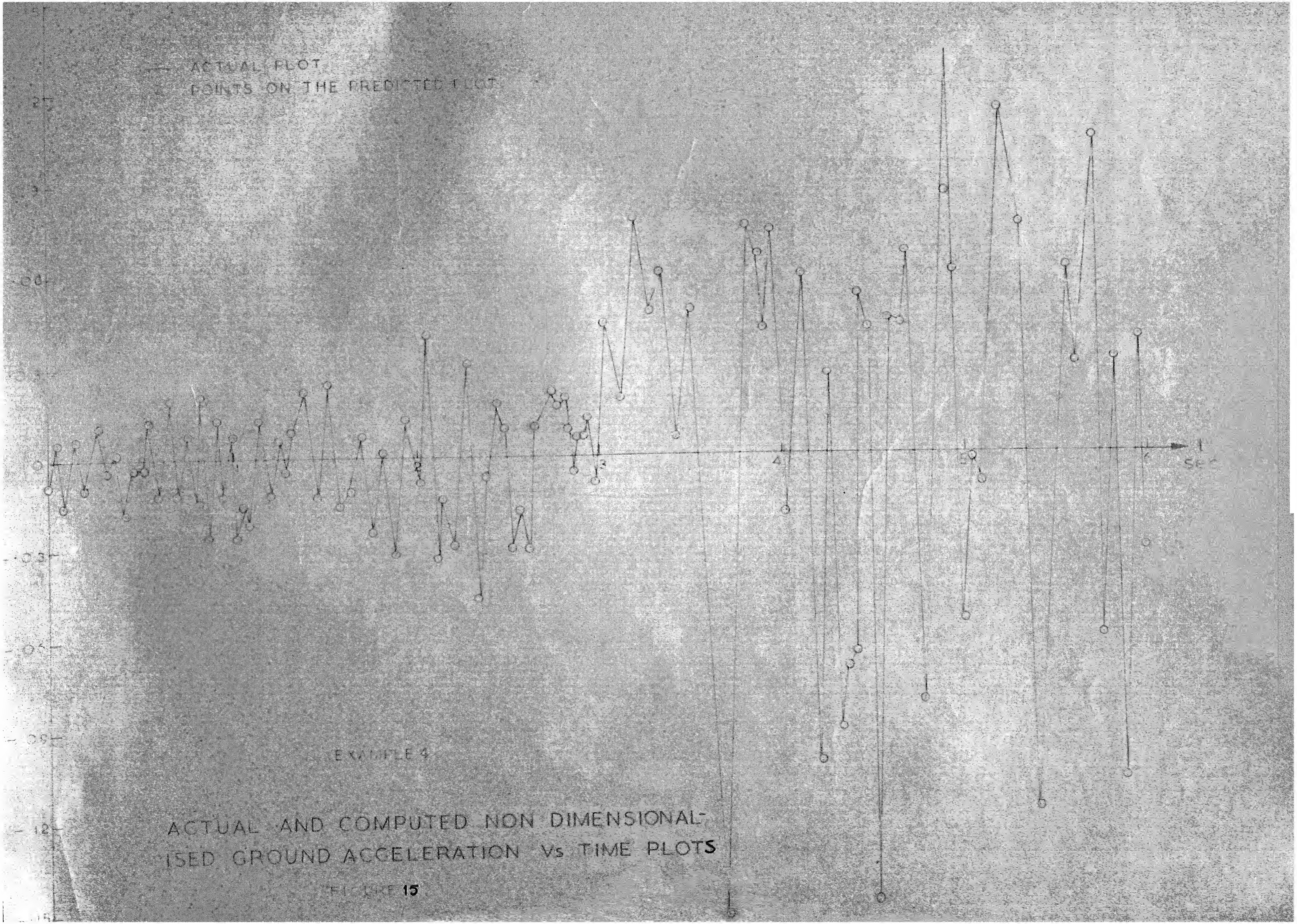
13



EXAMPLE 4

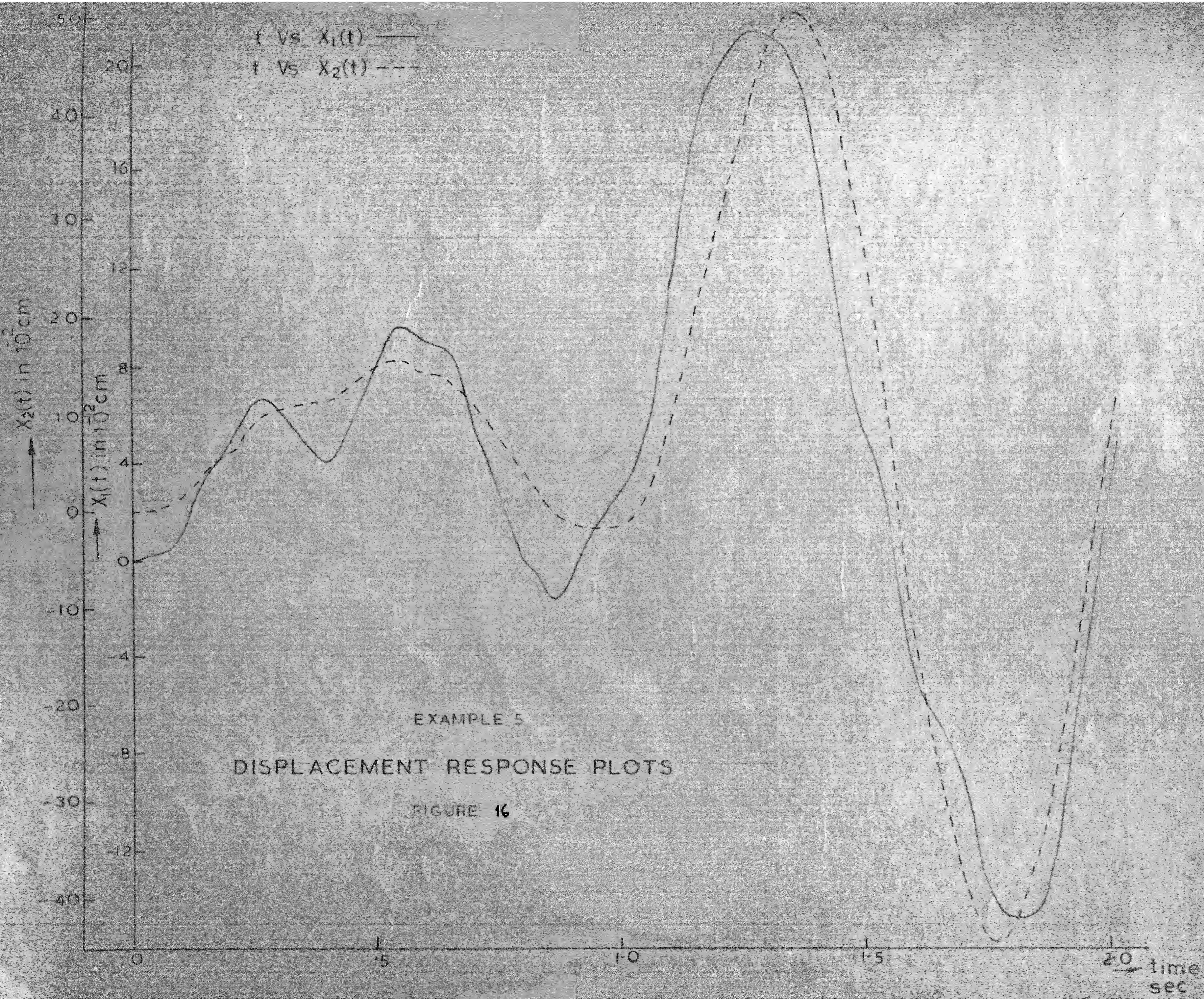
ACTUAL AND COMPUTED NON DIMENSIONALISED GROUND ACCELERATION Vs TIME PLOTS

FIGURE 14



ACTUAL AND COMPUTED NON DIMENSIONAL-
ISED GROUND ACCELERATION Vs TIME PLOTS

FIGURE 15



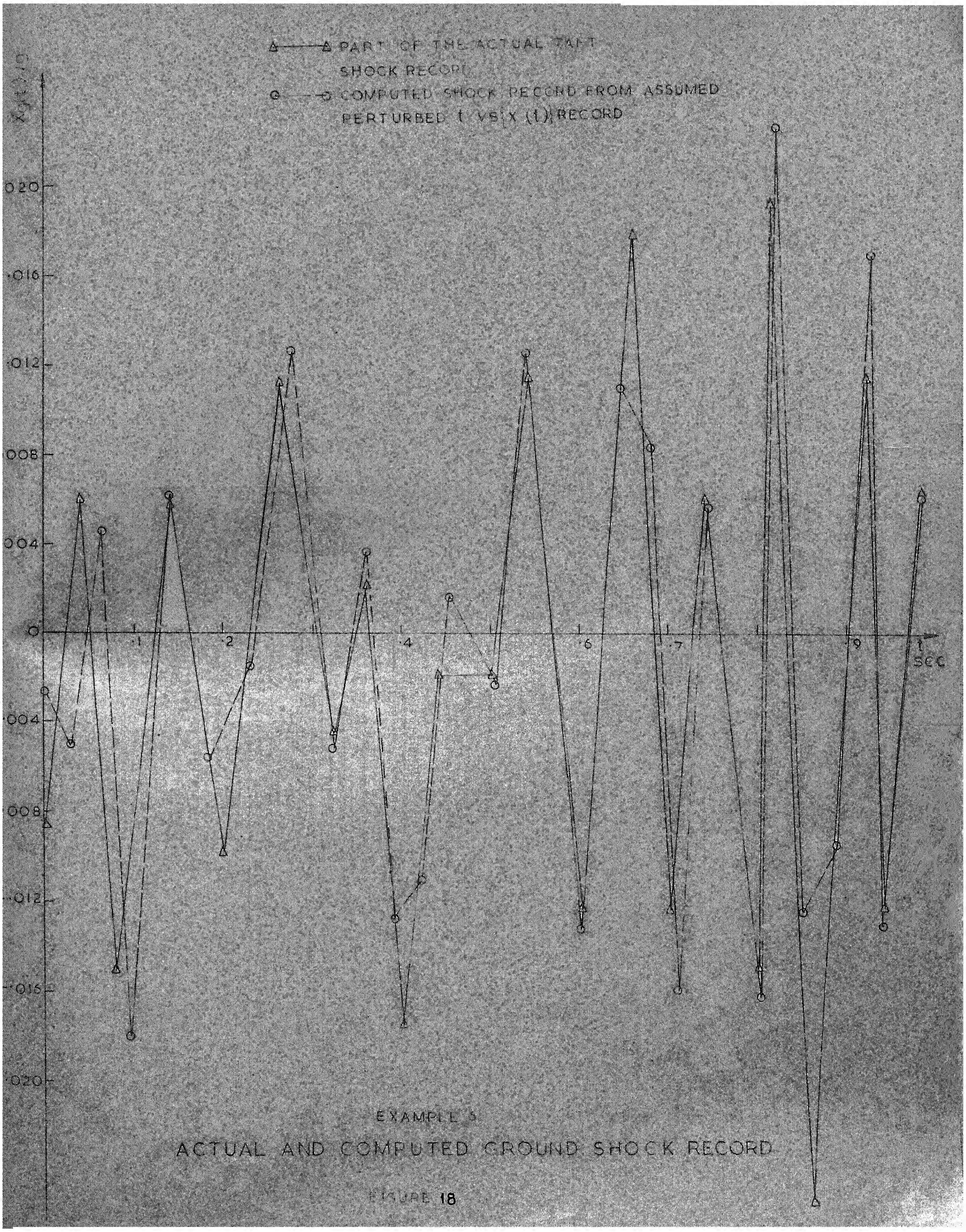
EXAMPLE 5

DISPLACEMENT RESPONSE PLOTS

FIGURE 16

EFFECT OF FRACTION OF DAMPING IN THE FIRST MODE
ON THE COMPUTED COORDINATES OF THE REQUIRED TIME
Vs NON-DIMENSIONALISED GROUND ACCELERATION PLOT

FIGURE 17



EXAMPLE 5

ACTUAL AND COMPUTED GROUND SHOCK RECORD

FIGURE 18

CHAPTER 4

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

4.1 Summary:

Certain algorithms have been developed to predict the time history of ground excitation of single and multi degree-of-freedom linear, damped systems from the known time history of the displacement or acceleration responses. The ground acceleration has been idealized as a continuous piecewise linear function over the entire time history. The inverse analysis of only the predominant mode is enough to predict accurately the time history of the ground acceleration for multi degree-of-freedom systems. The accuracy of basic collocation approach used in the present work depends on how close the digitization of the actual known response plots have been made.

4.2 Conclusions:

(1) The developed algorithms are numerically stable and successfully predict the time history of ground excitation of both the slow motion and strong motion ground shocks.

(2) Any local perturbation in displacement response records gets reflected in computation of ground accelerations, using DR-GA algorithm, in that local region. It does not bias prediction at a later stage.

(3) The DR-GA algorithm has been shown to be numerically better than the central finite difference method through rigorous error analysis.

(4) Perturbations in the value of fraction of critical damping in the fundamental mode do not seem to affect the prediction of ground shock using the algorithm DR-GA.

(5) The AR-GA algorithm is sensitive to the values of the known acceleration response parameter and influences the ground shock from the instance of perturbation onwards.

4.3 Recommendations:

In any effort of this kind, with time constraints, it is not possible to bring the work to its logical end. Hence it is recommended that the application of these algorithms to more complex structural systems be tried and the results compared.

The effect of base line shifts remains to be studied. However, it is expected that such a shift may materially affect the DR-GD and AR-GA algorithms as compared to DR-GA algorithm. This is because in the DR-GD and AR-GA algorithms terms involving double integrals are to be computed which are likely to be affected considerably in the later part of the time history because of greater value of time, t.

The preciseness of the analytical methods developed in the present work depends on the known response records and hence it is very important to improve upon the measuring devices i.e. displacement meters and accelerometers. Furthermore, they should be mounted on various structures in any susceptible ground motion belt.

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APPENDIX I

ERROR ANALYSIS OF STEP BY STEP METHOD

A step by step method, which is used for prediction of time history of displacement response of a structure subjected to ground excitation, has been tried for performing the inverse process. Numerical experimentations have shown that the method han not be used to advantage as any previous error, computational or otherwise, is carried through and its effect is quite severe. Without any loss of generality, an error analysis for a single degree-of-freedom system has been done to show how the method becomes sensitive to any small change.

The time history of displacement of a single degree-of-freedom undamped system with natural frequency, ω , subjected to the idealized ground acceleration, $\ddot{x}_g(t)$ shown in Fig. 2, is given by

$$\ddot{x}(t) + \omega^2 x(t) = -\ddot{x}_g(t) \qquad \text{(AI-1)}$$

For $t_i^* \leq t \leq t_{i+1}^*$:

$$\ddot{x}_g(t) = \ddot{x}_g(t_i^*) + b_i\,(t-t_i^*) \qquad \text{(AI-2)}$$

where

$$b_i = \frac{\ddot{x}_g(t_{i+1}^*) - \ddot{x}_g(t_i^*)}{t_{i+1}^*-t_i^*} \qquad \text{(AI-3)}$$

With initial conditions $x(t_i^*)$ and $\dot{x}(t_i^*)$, the displacement response is given by

$$x(t) = x(t_i^*)\cos\omega(t-t_i^*) + \frac{\dot{x}(t_i^*)}{\omega}\sin\omega(t-t_i^*) - \frac{\ddot{x}_g(t_i^*)}{\omega^2}\left[1-\cos\omega(t-t_i^*)\right] - \frac{b_i}{\omega^3}\left[\omega(t-t_i^*)-\sin\omega(t-t_i^*)\right] \quad \ldots \quad \text{(AI-4)}$$

Substituting equation (AI-3) in (AI-4) yields

$$x(t^*_{i+1}) = x(t^*_i)\cos\omega(t^*_{i+1}-t^*_i) + \frac{\dot{x}(t^*_i)}{\omega}\sin\omega(t^*_{i+1}-t^*_i)$$
$$-\frac{\ddot{x}_g(t^*_i)}{\omega^2}\left[-\cos\omega(t^*_{i+1}-t^*_i) + \frac{\sin\omega(t^*_{i+1}-t^*_i)}{\omega(t^*_{i+1}-t^*_i)}\right]$$
$$-\frac{\ddot{x}_g(t^*_{i+1})}{\omega^2}\left[1 - \frac{\sin\omega(t^*_{i+1}-t^*_i)}{\omega(t^*_{i+1}-t^*_i)}\right] \qquad \text{(AI-5)}$$

But, consideration of the previous linear segment in the idealized ground acceleration plot gives

$$\dot{x}(t^*_i) = -x(t^*_{i-1})\,\omega\sin\omega(t^*_i-t^*_{i-1}) + \dot{x}(t^*_{i-1})\cos\omega(t^*_i-t^*_{i-1})$$
$$+\ddot{x}_g(t^*_{i-1})\left[-\frac{\sin\omega(t^*_i-t^*_{i-1})}{\omega} + \frac{1-\cos\omega(t^*_i-t^*_{i-1})}{\omega^2(t^*_i-t^*_{i-1})}\right]$$
$$+\frac{\ddot{x}_g(t^*_i)}{\omega^2(t^*_i-t^*_{i-1})}\left[-1+\cos\omega(t^*_i-t^*_{i-1})\right] \qquad \text{(AI-6)}$$

Therefore, the coefficients of $\ddot{x}_g(t^*_i)$ and $\ddot{x}_g(t^*_{i+1})$ in equation (AI-5) are respectively given by

$$\ddot{x}_g(t^*_i) : R = \frac{1}{\omega^2}\left[\frac{\sin\omega(t^*_{i+1}-t^*_i)}{\omega(t^*_i-t^*_{i-1})}\left\{-1+\cos\omega(t^*_i-t^*_{i-1})\right\}\right.$$
$$\left.+\cos\omega(t^*_{i+1}-t^*_i) - \frac{\sin\omega(t^*_{i+1}-t^*_i)}{\omega(t^*_{i+1}-t^*_i)}\right] \qquad \text{(AI-7)}$$

$$\ddot{x}_g(t^*_{i+1}) : S = \frac{1}{\omega^2}\left[-1 + \frac{\sin\omega(t^*_{i+1}-t^*_i)}{\omega(t^*_{i+1}-t^*_i)}\right] \qquad \text{(AI-8)}$$

Use of equation (AI-5) for prediction of time history of ground acceleration requires knowledge of $\ddot{x}_g$ and $\dot{x}$ at the starting instant of the record. Further, the computed value of $\ddot{x}_g(t^*_{i+1})$ may be in error chiefly due to two causes: (i) digitization error in x(t), and (ii) error incurred during computation of previous ground accelerations.

In order that the digitization error in x(t) does not get magnified in computation of $\ddot{x}_g(t^*_{i+1})$, absolute value of S should be greater than 1, which is too harsh to be guaranteed in a physical problem.

Non magnification of errors in the computed $\ddot{x}_g(t^*_i)$ during computation of $\ddot{x}_g(t^*_{i+1})$ requires

$$\left|\frac{R}{S}\right| < 1$$

where $|\quad|$ denotes magnitude.

Approximations for sine and cosine terms given below are made so as to see meaning of the above constraint.

$$-1 + \frac{\sin\omega(t^*_{i+1}-t^*_i)}{\omega(t^*_{i+1}-t^*_i)} = -\frac{\omega^2(t^*_{i+1}-t^*_i)^2}{6} \qquad \text{(AI-9)}$$

$$-1 + \cos\omega(t^*_{i+1}-t^*_i) = -\frac{\omega^2(t^*_{i+1}-t^*_i)^2}{2} \qquad \text{(AI-10)}$$

$$-1 + \cos\omega(t^*_i-t^*_{i-1}) = -\frac{\omega^2(t^*_i-t^*_{i-1})^2}{2} \qquad \text{(AI-11)}$$

Thus,

$$\frac{R}{S} = 2+3\left(\frac{t^*_i-t^*_{i-1}}{t^*_{i+1}-t^*_i}\right)^2\left[\frac{t^*_{i+1}-t^*_i}{t^*_i-t^*_{i-1}} - \frac{\omega^2(t^*_{i+1}-t^*_i)^3}{6(t^*_i-t^*_{i-1})}\right] \qquad \text{(AI-12)}$$

For equal step size, $t^*_{i+1}-t^*_i = t^*_i-t^*_{i-1} = h$, the constraint $\left|\frac{R}{S}\right| < 1$ requires

$$\frac{\sqrt{8}}{\omega} < h < \frac{\sqrt{12}}{\omega}$$

i.e. $\frac{T}{2.2} < h < \frac{T}{1.8}$ (approximately)

where $T = \frac{2\pi}{\omega}$

Such a value for the stepsize is difficult to be guaranteed in a physical problem.

The equation (AI-12), thus, suggests that for a physical problem, the method as discussed above will not be successful in accurate prediction of time history of ground acceleration.

APPENDIX II

BERG - HOUSNER ALGORITHM

Purpose:

To compute ground displacement and velocity from the ground acceleration record.

Method:

This is based on a closed form integration of the idealized ground acceleration record consisting of a series of linear segments. For a linear segment from time t_i^* to t_{i+1}^*, the expressions given by Berg and Housner[13] are,

$$\dot{x}_g(t_{i+1}^*) = \dot{x}_g(t_i^*) + \frac{1}{2}(t_{i+1}^*-t_i^*)\left[\ddot{x}_g(t_i^*)+\ddot{x}_g(t_{i+1}^*)\right] \qquad \text{(AIII-1)}$$

$$x_g(t_{i+1}^*) = x_g(t_i^*) + \dot{x}_g(t_i^*)(t_{i+1}^*-t_i^*) + \frac{1}{6}(t_{i+1}^*-t_i^*)^2\left[2\ddot{x}_g(t_i^*)+\ddot{x}_g(t_{i+1}^*)\right] \qquad \text{(AIII-2)}$$

where the notations are as previously defined. Recursive use of equations (AIII-1) and (AIII-2) gives the required information. However, this requires the value of x_g and $\dot{x}_g$ at the start of the record which are usually taken to be zero.

APPENDIX III

SUBROUTINE TO COMPUTE DISPLACEMENT RESPONSE

Identification:

DVAGTS : Fortran IV Subroutine

Purpose:

Due to the absence of measured displacement response plot, this subroutine has been used to compute the displacement response of a physical system subjected to the known ground acceleration plot.

Method:

This is based on the approximation of the ground acceleration plot as a series of linear segments[5].

Explanation:

CALL DVAGTS (BETA,FREQ,TIMINI,TIMFIN,DISPIN,VELOIN,
DISPFN,EXCTIN,EXCTFN)

where

BETA	=	fraction of critical damping in the mode under consideration.
FREQ	=	natural frequency of the respective mode.
TIMINI	=	starting timeof a linear segment.
TIMFIN	=	end time of the same linear segment.
DISPIN	=	value of the displacement at time TIMINI.

VELOIN = value of the velocity at TIMINI, but after going through the execution stage of this subroutine, VELOIN will store value of velocity at time TIMFIN.

DISPFN = value of the displacement at time TIMFIN.

EXCTIN = value of ground acceleration at time TIMINI.

EXCTFN = value of ground acceleration at time TIMFIN.

Use:

This subroutine gives the value of displacement and velocity at time TIMFIN and its execution requires the knowledge of the parameters listed above except DISPFN.

```
[DEFIC DVACTS
      SUBROUTINE DVACTS(BETA,FREQ,TIMINI,TIMFIN,DISPIN,VELOIN,DISPFN,EXC
     1TIN,EXCTFN)
C***    SUBROUTINE   TO   COMPUTE   DISPLACEMENT   RESPONSE            ***
      DELTIM=TIMFIN-TIMINI
      DYFREQ=FREQ*SQRT(1.-BETA**2)
      BEFREQ=BETA*FREQ
      B1=(EXCTFN-EXCTIN)/DELTIM
      CONSC1=(BEFREQ*DISPIN+VELOIN-B1*(2.*BETA**2-1.)/(FREQ**2)+EXCTIN*B
     1ETA/FREQ)/DYFREQ
      CONSC2=DISPIN-2.*B1*BETA/(FREQ**3)+EXCTIN/(FREQ**2)
      EX1=EXP(-BEFREQ*DELTIM)
      FSIN=SIN(DYFREQ*DELTIM)
      FCOS=COS(DYFREQ*DELTIM)
      TEMPST=EX1*(CONSC1*FSIN+CONSC2*FCOS)
      DISPFN=TEMPST-EXCTFN/(FREQ**2)+2.*BETA*B1/(FREQ**3)
      VELOIN=-BEFREQ*TEMPST+DYFREQ*EX1*(CONSC1*FCOS-CONSC2*FSIN)-B1/(FRE
     1Q**2)
      RETURN
      END
```

APPENDIX IV

FLOW CHART

<u>Explanation:</u>

This appendix gives the salient features of the steps to be taken while writing the computer programme to find ground acceleration from displacement or acceleration response record.

Start

↓

Read System Properties

↓

Set up the Mass And Stiffness Matrices

↓

Call Subroutine EIGPRM to get the Flexibility Formulation of the Eigenvalue Problem

↓

Call Subroutine Eigen To get the Fundamental Natural Frequency and the Associated Mode Shape

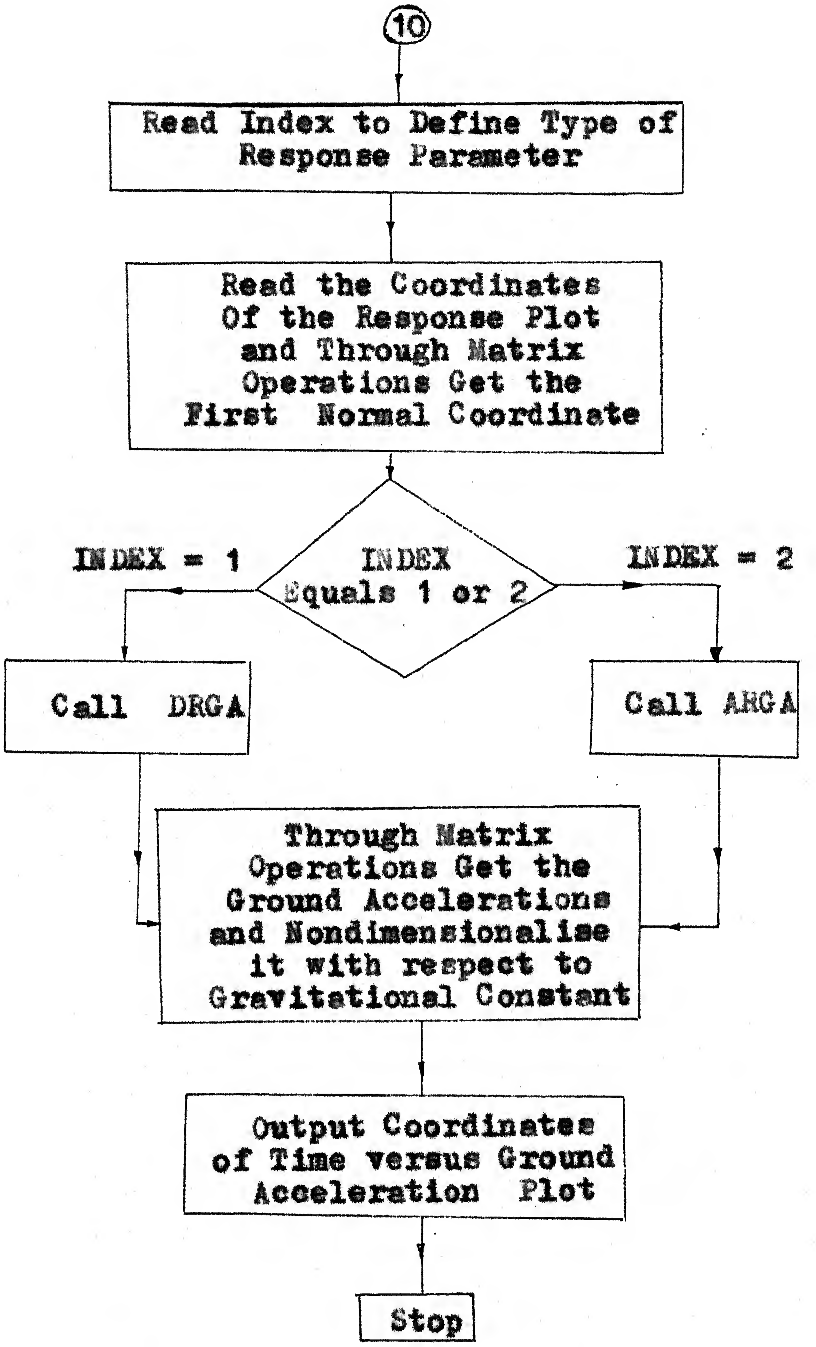
↓

Normalise the Computed Mode Shape with respect to Mass Matrix

↓

Output Fundamental Natural Frequency and the Associated Normalised Mode Shape

↓


10
Read Index to Define Type of Response Parameter
Read the Coordinates Of the Response Plot and Through Matrix Operations Get the First Normal Coordinate
INDEX Equals 1 or 2
INDEX = 1
INDEX = 2
Call DRGA
Call ARGA
Through Matrix Operations Get the Ground Accelerations and Nondimensionalise it with respect to Gravitational Constant
Output Coordinates of Time versus Ground Acceleration Plot
Stop

APPENDIX V

COMPUTER PROGRAMME

Object:

To predict Ground Acceleration from displacement or acceleration response record.

Identification:

MAIN,DRGA,ARGA,EIGPRM,MATMUL,EIGEN,MATVEC,GAJORD

- Fortran IV Subroutines

Explanation of Fortran Variables:

USED AS	No.	Variable	Explanation
INPUT	1	NDOF	Number of degrees of freedom of the idealized structure.
	2	GRAVTC	Gravitational Constant.
	3	MOFST(I)	Mass lumped at the level of I-th coordinate.
	4	KOFST(I)	Stiffness of the elastic element connecting masses lumped at (I-1) th and I-th level.
	5	INDEX	In order to use algorithm DRGA or ARGA, INDEX is respectively assigned 1 or 2.
	6	NTP	Number of digitized time coordinates in the response plot.

Explanation of Fortran Variables: (Contd.)

USED AS	No.	Variable	Explanation
INPUT (Contd.)	7	NDTP1C	Number of data points defined on one computer card. NDTP1C = 1 in case more cards are used to define one data point.
	8	ERRTOL	Tolerance for the relative error in response to find out approximate range of a linear segment in the excitation plot. It is to be given in order to use DR-GA algorithm.
	9	GENDIS	Initial value of Normal Coordinate (Assigned zero in case of structure starting from rest). It is to be given in order to initiate the algorithm AR-GA.
	10	GENVEL	Initial value of time derivative of normal coordinate (Assigned zero in case of structure starting from rest). It is to be given in order to initiate the algorithm AR-GA.
	11	A	Defines the time coordinates of the data points of the response plot.
	12	B	Defines the response coordinates of the data points of the response plot.

Explanation of Fortran Variables: (Contd.)

USED AS	No.	Variable	Explanation
OUTPUT	13.	MASS	Assembled mass matrix for the structure.
	14	STIFF	Assembled stiffness matrix for the structure.
	15	FUFREQ	Fundamental natural frequency of the structure.
	16	FUMODE	Fundamental mode shape.
	17	EXCTN	Vector of ground acceleration coordinates of the desired ground acceleration plot.
	18	TIME1 (if INDEX=1) or TIME (if INDEX=2)	Vector of time coordinates of the desired ground acceleration plot.
INPUT and OUTPUT	19	FRDAMP	Fraction of critical damping in the considered mode.

```
$IBFTC MAIN
      DIMENSION A(5),B(10,5),FUNMAS(10)
      REAL MOFST(10),KOFST(10),MASS(10,10)
      COMMON/LBL0/FUMODE(10)
      COMMON/LBL1/FRDAMP,FUFREQ
      COMMON/LBL2/RESPNS(1,2000),TIME(2000),ERRTOL,NTP
      COMMON/LBL3/EXCTN(2000),TIME1(500),NCHPTS
      COMMON/LBL4/MASS,STIFF(10,10)
      COMMON/LBL5/DYNMAT(10,10),NDOF
      COMMON/LBL6/GENDIS,GENVEL
      COMMON/MULT/VECTOR(10)
C     *** ASSIGN  INDEX=1  IN CASE OF DISPLACEMENT RESPONSE              ***
C     *** ASSIGN  INDEX=2  IN CASE OF ACCELERATION RESPONSE              ***
 1010 FORMAT(2I5)
 1020 FORMAT(2F10.4)
 1030 FORMAT(2(F10.4,2E15.8))
 2010 FORMAT(X,130(*-*))
 2020 FORMAT(6X,*MASS MATRIX*)
 2030 FORMAT(6X,*STIFFNESS MATRIX*)
 2040 FORMAT(6X,*TRANSPOSE OF FUNDAMENTAL MODE SHAPE*)
 2050 FORMAT(6X,*COORDINATES OF TIME VERSUS EXCITATION PLOT*)
 3010 FORMAT(X,6E20.8)
 3020 FORMAT(6X,*FUNDAMENTAL NATURAL FREQUENCY =*,X,E15.8,X,*RADIANS PER
     1 UNIT SECOND*)
 3030 FORMAT(4(F10.4,E20.8))
C     *** FEEDING  OF  THE  SYSTEM  CHARACTERISTICS                      ***
      READ  (5,1010) NDOF
      READ  (5,1020) GRAVTC
      READ  (5,1020) (MOFST(I),KOFST(I),I=1,NDOF)
C     *** MASS MATRIX                                                    ***
      DO 120 I=1,NDOF
      DO 110 J=1,NDOF
  110 MASS(I,J)=0.0
  120 MASS(I,I)=MOFST(I)
C     *** STIFFNESS  MATRIX                                              ***
      DO 150 I=1,NDOF
      DO 130 J=1,I
  130 STIFF(I,J)=0.0
      IF(I.EQ.1) GO TO 140
      STIFF(I,I-1)=-KOFST(I)
  140 STIFF(I,I)=KOFST(I)
      IF(I.EQ.NDOF) GO TO 150
      STIFF(I,I)=STIFF(I,I)+KOFST(I+1)
  150 CONTINUE
      DO 160 I=1,NDOF
      DO 160 J=I,NDOF
  160 STIFF(I,J)=STIFF(J,I)
C     *** COMPUTATION  OF  FUNDAMENTAL  NATURAL  FREQUENCY  AND          ***
C     *** MODE  SHAPE                                                    ***
      CALL EIGPRM
```

```
      CALL EIGEN
C     *** NORMALISATION OF FUNDAMENTAL MODE                                  ***
      SCALAR=0.0
      DO 180 I=1,NDOF
      FUNMAS(I)=0.0
      DO 170 J=1,NDOF
  170 FUNMAS(I)=FUNMAS(I)+FUMODE(J)*MASS(J,I)
  180 SCALAR=SCALAR+FUMODE(I)*FUNMAS(I)
      SCALAR=SQRT(SCALAR)
      DO 190 I=1,NDOF
      FUNMAS(I)=FUNMAS(I)/SCALAR
  190 FUMODE(I)=FUMODE(I)/SCALAR
C     *** LISTING OF THE SYSTEM CHARACTERISTICS                              ***
      WRITE(6,2010)
      WRITE(6,2020)
      WRITE(6,1020) ((MASS(I,J),J=1,NDOF),I=1,NDOF)
      WRITE(6,2010)
      WRITE(6,2030)
      WRITE(6,1020)((STIFF(I,J),J=1,NDOF),I=1,NDOF)
      WRITE(6,2010)
      WRITE(6,3020) FUFREQ
      WRITE(6,2040)
      WRITE(6,3010) (FUMODE(I),I=1,NDOF)
      WRITE(6,2010)
      *** DATA FEEDING AND UNCOUPLING THE EQUATIONS                          ***
      READ (5,1010) INDEX,NTP,NDTP1C
      READ (5,1020) FRDAMP
      WRITE(6,3010) FRDAMP
      GO TO (1910,1920),INDEX
 1910 READ(5,3010) ERRTOL
      GO TO 195
 1920 READ(5,3010) GENDIS,GENVEL
  195 CONTINUE
      NTEMP=NDTP1C
      K=0
  200 K=K+1
      I=NDTP1C*(K-1)
      IF((I+NDTP1C).GT.NTP)NTEMP=NTP-I
      READ (5,1030) (A(J),(B(M,J),M=1,NDOF),J=1,NTEMP)
      DO 210 J=1,NTEMP
      II=I+J
      TIME(II)=A(J)
      RESPNS(1,II)=0.0
      DO 210 M=1,NDOF
  210 RESPNS(1,II)=RESPNS(1,II)+B(M,J)*FUNMAS(M)
      IF(II.LT.NTP) GO TO 200
      GO TO(2210,2220),INDEX
 2210 CALL DRGA
      GO TO 230
 2220 CALL ARGA
```

```
      A(2,3)=TIME(I+3)-TIME(I)
      A(2,4)=DISP(IEQN,I+3)-DISP(IEQN,I)
      A(3,1)=-1.0+EX2COS
      A(3,2)=EX2SIN
      A(3,3)=TIME(I+2)-TIME(I)
      A(3,4)=DISP(IEQN,I+2)-DISP(IEQN,I)
      CALL GAJORD(A,3)
      C4I=DISP(IEQN,I)-A(1,1)+2.0*A(3,1)*BETA/FREQ
   10 J=J+1
      IF(J.GT.(NTP-3))NEND=1
      IF(J.GT.NTP)  GO TO 15
      DIS =EXP(-BEFREQ*(TIME(J)-TIME(I)))*(A(1,1)*COS(DYFREQ*(TIME(J)-TI
     1ME(I)))+A(2,1)*SIN(DYFREQ*(TIME(J)-TIME(I))))+A(3,1)*(TIME(J)-TIME
     2(I)-2.*BETA/FREQ)+C4I
      ERRALL=ERRTOL*ABS(DISP(IEQN,J))
      IF(ERRALL.EQ.0.0)ERRALL=ERRTOL
      IF(ABS(DIS-DISP(IEQN,J)).LE.ERRALL) GO TO 10
      IF(NSTART.EQ.1) GO TO 45
      IF(I.EQ.1) GO TO 25
   15 TIME1(M)=(A(3,1)*TIME(I)-TEMP*TIME(IOLD)+TEMP1-C4I)/(A(3,1)-TEMP)
      IF(TIME1(M).LE.TIME(I-1).OR.TIME1(M).GE.TIME(I)) GO TO 30
      EXCTN(M)=-SOMEG2*(C4I+A(3,1)*(TIME1(M)-TIME(I)))
   45 NSTART=2
      IF(NEND.EQ.1) GO TO 35
      TEMP=A(3,1)
      TEMP1=C4I
      IOLD=I
      I=J
      GO TO 20
   25 TEMP=A(3,1)
      TEMP1=C4I
      TIME1(M)=TIME(I)
      EXCTN(M)=-SOMEG2*(C4I+TEMP*(TIME1(M)-TIME(I)))
      IOLD=I
      I=J
      GO TO 20
   30 TIME1(M)=TIME(I-1)
      EXCTN(M)=-SOMEG2*(TEMP1+TEMP*(TIME1(M)-TIME(IOLD)))
      NSTART=1
      I=I-1
      GO TO 21
   35 M=M+1
      TIME1(M)=TIME(J-1)
      EXCTN(M)=-SOMEG2*(C4I+A(3,1)*(TIME1(M)-TIME(I)))
      RETURN
      END
```

```
$IBFTC ARGA
      SUBROUTINE ARGA
C     ******************************************************************
C     *        ACCELERATION  IS  THE  KNOWN  RESPONSE  PARAMETER       *
C     ******************************************************************
      COMMON/LBL1/BETA,FREQ
      COMMON/LBL2/ACCL(1,2000),TIME(2000),ERRTOL,NTP
      COMMON/LBL3/EXCTM(2000),TIME1(500),M
      COMMON/LBL6/GENDIS,GENVEL
      VINTG1(FREQ,BETA,SQRBET,EX1,FCOS,FSIN)=(BETA*(1.0-EX1*FCOS)+SQRBET
     1*EX1*FSIN)/FREQ
      VINTG2(FREQ,BETA,SQRBET,EX1,FCOS,FSIN)=(-BETA*EX1*FSIN+SQRBET*(1.0
     1-EX1*FCOS))/FREQ
      M=NTP
      IEQN=1
      BEFREQ=BETA*FREQ
      SOMEG2=FREQ**2
      SQRBET=SQRT(1.-BETA**2)
      DYFREQ=FREQ*SQRBET
      EXCTM(1)=-(ACCL(IEQN,1)+2.*BEFREQ*GENVEL+SOMEG2*GENDIS)
      DO 100 I=2,M
      DELTIM=TIME(I)-TIME(I-1)
      EX1=EXP(-BEFREQ*DELTIM)
      FCOS=COS(DYFREQ*DELTIM)
      FSIN=SIN(DYFREQ*DELTIM)
      CONSC=ACCL(IEQN,I-1)
      CONSD=(ACCL(IEQN,I)/EX1-ACCL(IEQN,I-1)*FCOS)/FSIN
      CONST1=(CONSC*BETA+CONSD*SQRBET)/FREQ
      CONST2=(CONSC*SQRBET-CONSD*BETA)/FREQ
      TEMP1=VINTG1(FREQ,BETA,SQRBET,EX1,FCOS,FSIN)
      TEMP2=VINTG2(FREQ,BETA,SQRBET,EX1,FCOS,FSIN)
      GENDIS=GENDIS+DELTIM*(GENVEL+CONST1)-CONST1*TEMP1+CONST2*TEMP2
      GENVEL=GENVEL+CONSC*TEMP1+CONSD*TEMP2
      EXCTM(I)=-(ACCL(IEQN,I)+2.*BEFREQ*GENVEL+SOMEG2*GENDIS)
  100 CONTINUE
      RETURN
      END
$IBFTC EIGPRM
      SUBROUTINE EIGPRM
      REAL MASS(10,10)
      COMMON/LBL4/MASS,STIFF(10,10)
      COMMON/LBL5/DYNMAT(10,10),NDOF
      DIMENSION A(10,11)
      DO 110 I=1,NDOF
      DO 100 J=1,NDOF
  100 A(I,J)=STIFF(I,J)
  110 A(I,NDOF+1)=1.0
      N=NDOF
      CALL GAJORD(A,N)
      DO 120 I=1,NDOF
```

```
      DO 120 J=1,NDOF
  120 A(I,J)=A(I,J+1)
      CALL MATMUL(A)
      RETURN
      END
$IBFTC MATMUL
      SUBROUTINE MATMUL(A)
      REAL MASS(10,10)
      COMMON/LBL4/MASS,STIFF(1),10)
      COMMON/LBL5/DYNMAT(10.10),NDOF
      DIMENSION A(10,11)
      DO 1 I=1,NDOF
      DO 1 J=1,NDOF
      DYNMAT(I,J)=0.0
      DO 1 K=1,NDOF
    1 DYNMAT(I,J)=DYNMAT(I,J)+A(I,K)*MASS(K,J)
      RETURN
      END
$IBFTC EIGEN
      SUBROUTINE EIGEN
      COMMON/LBL0/X(10)
      COMMON/LBL1/BETA,FREQLW
      COMMON/LBL5/C(10,10),NDOF
      COMMON/MULT/Y(10)
      LOGICAL Q
      NN=NDOF-1
      TOLRCE=0.1E-05
      DO 100 I=1,NDOF
  100 X(I)=1.0
   10 CALL MATVEC
      Q=.FALSE.
      IF(Y(NDOF).EQ.0.0) GO TO 3
      DO 1 I=1,NN
      Y(I)=Y(I)/Y(NDOF)
    1 CONTINUE
      Y(NDOF)=1.0
    3 DO 2 I=1,NDOF
      IF(ABS(Y(I)-X(I)).GT.TOLRCE) Q=.TRUE.
      IF(Q) GO TO 30
    2 CONTINUE
   30 DO 4 I=1,NDOF
      X(I)=Y(I)
    4 CONTINUE
      IF(Q) GO TO 10
      CALL MATVEC
      SUM=0.0
      SUM1=0.0
      DO 5 I=1,NDOF
      SUM=SUM+X(I)**2
      SUM1=SUM1+X(I)*Y(I)
```

```
    5 CONTINUE
      FREQLN=SQRT(SUM/SUM1)
      RETURN
      END
$IBFTC MATVEC
      SUBROUTINE MATVEC
      COMMON/LBL0/X(10)
      COMMON/LBL5/C(10,10),NDOF
      COMMON/MULT/Y(10)
      DO 2 I=1,NDOF
      Y(I)=0.0
      DO 1 J=1,NDOF
      Y(I)=Y(I)+C(I,J)*X(J)
    1 CONTINUE
    2 CONTINUE
      RETURN
      END
$IBFTC GAJORD
      SUBROUTINE GAJORD(A,N)
      DIMENSION A(10,11),B(11)
      M=N+1
   50 DO 100 K=1,N
      DO 60 I=2,M
   60 B(I-1)=A(1,I)/A(1,1)
      B(M)=1.0/A(1,1)
      L=N-1
      LS=M-1
      DO 80 I=1,L
      DO 70 J=1,LS
   70 A(I,J)=A(I+1,J+1)-B(J)*A(I+1,1)
      A(I,M)=-B(M)*A(I+1,1)
   80 CONTINUE
      DO 90 I=1,M
   90 A(N,I)=B(I)
  100 CONTINUE
      RETURN
      END
```

SAMPLE INPUT

```
  2
980.0
  1.0        75.0
  1.0        50.0
  1  1921     2
  0.0
     .1         E-05
  0.0000 0.                     0.                   0.0100 0.30864228E-03 0.37939300E-
  0.0200 0.10678828E-02 0.12465684E-02     0.0300 0.19432205E-02 0.22361298E-
  0.0400 0.25757941E-02 0.29944099E-02     0.0500 0.27528822E-02 0.33120033E-
  0.0600 0.28420876E-02 0.35567031E-02     0.0700 0.33560073E-02 0.42405418E-
```